SOLUTION OF PREPARATORY PROBLEMS

for the

First International Nuclear Science Olympiad (1st INSO)

Solution of 1st INSO Preparatory Problems

Prepared by the 1st INSO International Jury

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PREFACE

We are pleased to present the solution of preparatory problems for the first International Nuclear Science Olympiad (INSO). The solution has been prepared by the INSO International Jury, a distinguished panel of experts from eight countries, organized by the International Atomic Energy Agency (IAEA).

Nuclear science plays a crucial role in advancing our understanding of the fundamental principles that govern the universe, as well as in addressing real-world challenges, from energy production to medical diagnostics. INSO aims to inspire and challenge the next generation of scientists who will contribute to the ever-evolving landscape of nuclear research.

We hope that this booklet can aid you in delving into the core principles and applications of nuclear science. Approach each question with curiosity and determination. These problems go beyond mere assessments, aiming to inspire creative thinking and a nuanced understanding of nuclear science concepts. This preparatory experience aims to both challenge and inspire, fostering a deeper appreciation for the complexities of atomic and subatomic phenomena.

We hope for your successful preparation for the competition and may your INSO experience inspire a lifelong passion for the captivating world of nuclear science.

First INSO International Jury

Q1. CREATION OF NEW PARTICLES (10 pts)

Part 1. Total Energy E (0.5 pts)

1. Total energy is equal to kinetic energy plus rest mass energy.

$$E = K + mc^2$$

1.	$E = K + mc^2$	0.5 pts

2.

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$m^2 = \frac{m_0^2}{(1 - v^2/c^2)}$$

$$c^4\left(1 - \frac{v^2}{c^2}\right)$$

$$m^2c^4\left(1 - \frac{v^2}{c^2}\right) = m_0^2c^4 \quad \Longrightarrow \quad m^2c^4 - m^2v^2c^2 = m_0^2c^4$$

$$p = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}, E = mc^2 \quad \text{and } E_0 = m_0c^2$$

$$E^2 = E_0^2 + (pc)^2$$

Hence $E^2 = p^2c^2 + m^2c^4$

2.	$E^2 = p^2c^2 + m^2c^4$	1.0 pts

3.

You might think that the creation of a new product particle, nine times more massive than in a previous experiment, would require just nine times more energy for the incident proton. Unfortunately not all of the kinetic energy K of the incoming proton is available to create the product particle, since conservation of momentum requires that after the collision the system as a whole still must have some kinetic energy. Only a fraction of the energy of the incident particle is thus available to create a new particle.

Solving the problem, we will use a relativistic expression for the relationship between the total energy of the incident proton E and its linear momentum p because accelerators of any type make particles moving at very high speeds. The mass of the proton is denoted as m_p. For two colliding protons as the system, where one of the particles is initially at rest, before the collision we have:

$$E_1 = K + m_p c^2 \qquad E_2 = m_p c^2$$
$$E_1^2 = p_1^2 c^2 + m_p^2 c^4 \qquad p_2 = 0$$

In the final state

$$E_f = K_f + Mc^2$$
$$E_f^2 = p_f^2 c^2 + M^2 c^4$$

By energy conservation

$$E_1 + E_2 = E_f$$
$$E_1^2 + 2E_1 E_2 + E_2^2 = E_f^2$$

$$p_1^2 c^2 + m_p^2 c^4 + 2(K + m_p c^2) m_p c^2 + m_p^2 c^4 = p_f^2 c^2 + M^2 c^4$$

By conservation of momentum $p_1 = p_f$

The last two equations give energy available to create a product particle as

$$Mc^2 = 2m_p c^2 \sqrt{1 + \frac{K}{2m_p c^2}}$$

From this result, when the kinetic energy K of the incident proton is large compared to its rest energy $m_p c^2$, we see that M approaches $(2m_p K)^{\frac{1}{2}}/c$.

Thus if the energy of the incoming proton is increased by a factor of nine, the mass you can create increases only by a factor of three.

3.	Thus if the energy of the incoming proton is increased by a factor of nine, the mass you can create increases only by a factor of three.	4.0 pts

4.

This disappointing result is the main reason that most modern accelerators, such as those at CERN (in Europe), at Fermilab (near Chicago), at SLAC (at Stanford), and at DESY (in Germany), use colliding beams. Here the total momentum of a pair of interacting particles can be zero. The center of mass can be at rest after the collision, so in principle all of the initial kinetic energy can be used for particle creation, according to

$$Mc^2 = mc^2 + \frac{K}{2} + mc^2 + \frac{K}{2} = 2mc^2 + K = 2mc^2\left(1 + \frac{K}{2mc^2}\right)$$

where K is the total kinetic energy of two identical colliding particles. Here if $K \gg mc^2$, we have M directly proportional to K, as we would desire.

These machines are difficult to build and to operate, but they open new vistas in physics.

4.	Here if $K \gg mc^2$, we have M directly proportional to K, as we would desire. These machines are difficult to build and to operate, but they open new vistas in physics.	3.5 pts

5.

Efficiency is estimated as a ratio of the consumed energy and the energy pumped into a device. This ratio is higher for the collider.

5.	The ratio is higher for the collider.	1.0 pts

Q2. CRITICALITY OF A REACTOR (10 pts)

1.

$$k = \frac{\text{Total Production Rate}}{\text{Total Consumption Rate } (Leakage + Absorptio)}$$

$$k = \frac{\nu\Sigma_f\phi}{DB^2\phi + \Sigma_a\phi} = \frac{\nu\Sigma_f}{DB^2 + \Sigma_a}$$

1.	$k = \frac{\nu\Sigma_f}{DB^2 + \Sigma_a}$	1.0 pts

2.

$$N({}^{235}_{92}U) = \frac{30\times19.1\times0.6022\times10^{24}}{100\times235.0439} = 0.014680683731\times 10^{24}\frac{Atoms}{cm^3}$$
$$= 1.4680683731\times10^{22}\ Atoms/cm^3$$

$$N({}^{238}_{92}U) = \frac{70\times19.1\times0.6022\times10^{24}}{100\times238.0508} = 0.03382225\times10^{24}\frac{Atoms}{cm^3}$$
$$= 3.382225\times10^{22}\frac{Atoms}{cm^3}$$

2.	$N({}^{235}_{92}U) = 1.468\times10^{22}\ Atoms/cm^3$ $N({}^{238}_{92}U) == 3.38\times10^{22} Atoms/cm^3$	2.0 pts

3.

$$D = \frac{1}{3\Sigma_{tr}} = \frac{1}{3[0.01468\times6.8 + 0.0338\times6.9]} = \frac{1}{0.999606} = 1.000394\ cm$$

$$B = \frac{\pi}{R} = \frac{3.1415}{100} = 0.031415\ cm^{-1}$$

$${}^{235}\Sigma_f = {}^{235}(N\sigma_f) = 0.01468\times1.4 = 0.020552\ cm^{-1}$$

$$\Sigma_a = {}^{235}(N\sigma_a) + {}^{238}(N\sigma_a)$$

$$\Sigma_a = 0.01468\times1.65 + 0.03382\times0.255 = 0.0328461\ cm^{-1}$$

$${}^{235}k = \frac{2.6\times0.020552}{(1.000394)(0.031415)^2 + 0.0328461} = \frac{0.0534352}{0..338334} = 1.57936$$

$${}^{238}\Sigma_f = 0.0338\times0.095 = 3.2131\times 10^{-3}\quad cm^{-1}$$

$$^{235+238}k = \frac{2.6(0.020552 + 0.003431}{0.0338334} = \frac{0.061789295}{0..338334} = 1.82628$$

3.	$^{235}k = 1.579$ $^{235+238}k = 1.826$	2.0 pts

4.

$$\text{Total Energy} = TE = E_R + kE_R + k^2E_R + \cdots + k^nE_R = (1 + k + k^2 + \cdots + k^n)E_R$$
$$= E_R\left(\frac{k^{n+1} - 1}{k - 1}\right)$$

$$\text{Energy in first (n − m) fissions} = E_R\left(\frac{k^{n-m+1} - 1}{k - 1}\right)$$

$$\text{Energy in final m fission events} = E_R\left[\frac{k^{n+1} - 1}{k - 1} - \frac{k^{n-m+1} - 1}{k - 1}\right]$$

$$\text{Energy in final m fission events} = E_R\left[\frac{k^{n+1} - 1 - k^{n-m+1} + 1}{k - 1}\right]$$

$$F_m = \text{Fraction of energy in final m generations} = E_R\left[\frac{k^{n+1} - k^{n-m+1}}{k^{n+1} - 1}\right]$$

4.	$F_m = \text{Fraction of energy in final m generations}$ $= E_R\left[\frac{k^{n+1} - k^{n-m+1}}{k^{n+1} - 1}\right]$	2.5 pts

5.

If n is large $F = F_m = E_R\left[\frac{1-k^{-m}}{1-1/k^{n+1}}\right]$

Now if $k > 1$ *and* $n \to \infty$, *then* $(\frac{1}{k^{n+1}}) \to 0$

$$F = 1 - k^{-m}$$

Now if $F = 99.9\% = 0.999$

$$0.999 = 1 - k^{-m}$$

$$k^{-m} = 0.001$$

Taking log on both sides

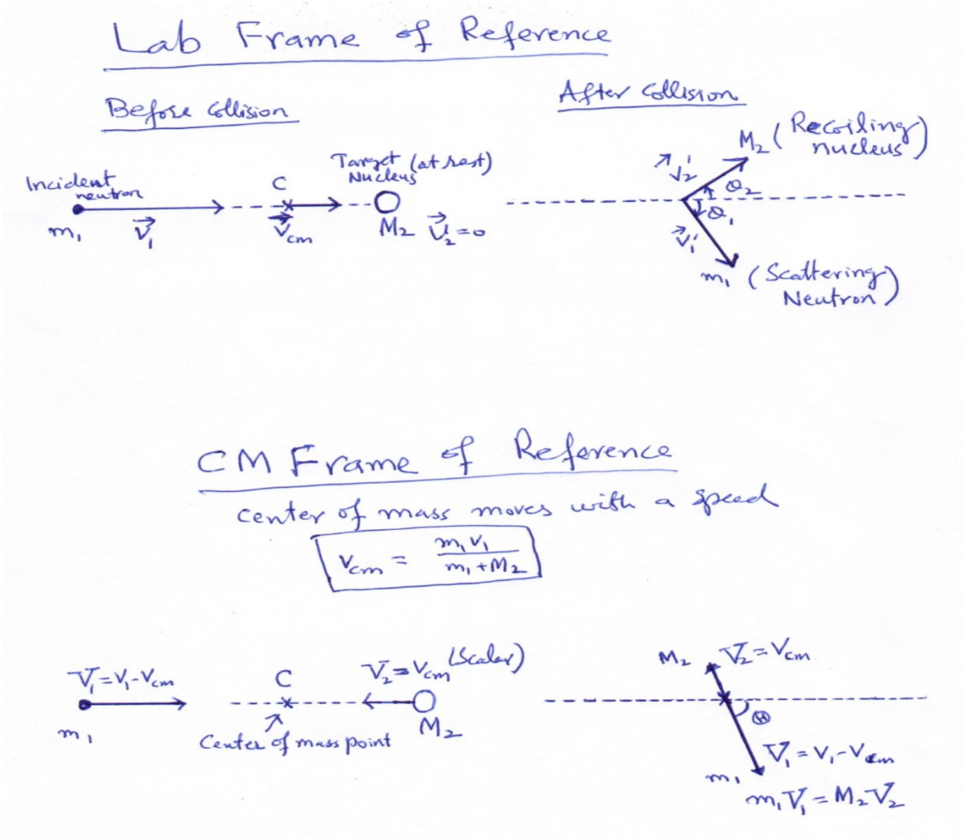
$$m = \frac{-\ln(0.001)}{\ln(k)} = \frac{6.90777}{\ln(1.82628)} = \frac{6.90777}{0.60228} = 11.469 \approx 12$$

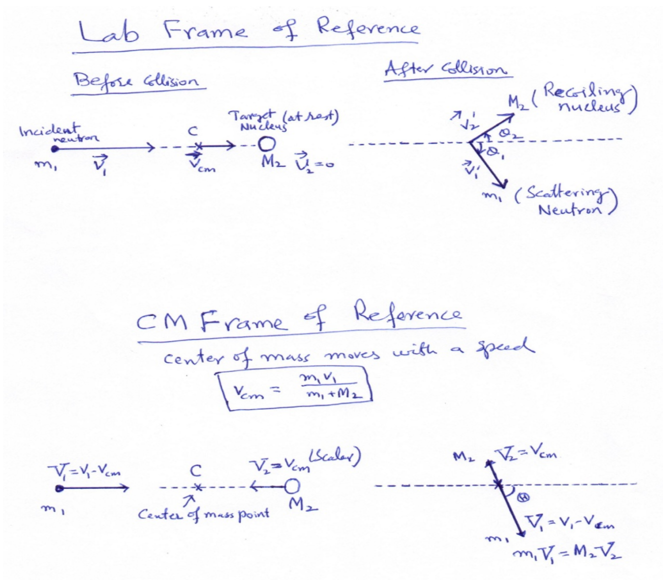
It means that 99.9% of the energy is released in 12×10^{-8} seconds.

5.	$m = 12$ 12×10^{-8} seconds	2.5 pts

Q3. ENERGY LOSS OF NEUTRON (10 pts)

1.



1.		1.0 pts
		

2.

$$v_{cm} = \frac{m_1 v_1}{m_1 + M_2}$$
$$\vec{V}_1 = \vec{v}_1 - \vec{v}_{cm}$$
$$\vec{V}_2 = -\vec{v}_{cm}$$

v_1 is the speed of neutron in the Lab frame of reference before the collision.
v_{cm} is the speed of the center of mass in the Lab frame of reference
V_1 is the speed of neutron in the CM frame of reference before and after the collision.
V_2 is the speed of nucleus in the CM frame of reference before and after the collision.

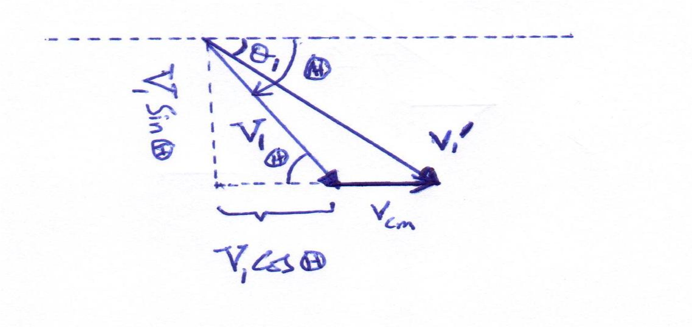
2.	$\vec{V}_1 = \vec{v}_1 - \vec{v}_{cm}$ $\vec{V}_2 = -\vec{v}_{cm}$	0.5 pts

3.
The total momentum in the CM frame of reference before and after the collision is zero. i.e., $m_1\vec{V}_1 + M_2\vec{V}_2 = 0$.

3.	The total momentum in the CM frame of reference before and after the collision is zero	0.5 pts

4.

Let us see the velocity of the neutron in both the frames of reference as seen in the Lab Frame as shown in figure below



From the bigger triangle we can write

$$(v_1')^2 = (V_1 \cos\Theta + v_{cm})^2 + (V_1 \sin\Theta)^2$$
$$(v_1')^2 = V_1^2\, cos^2\Theta + v_{cm}^2 + 2V_1 v_{cm}\cos\Theta + V_1^2\, sin^2\Theta$$
$$(v_1')^2 = V_1^2 + v_{cm}^2 + 2V_1 v_{cm}\cos\Theta$$

4.	$(v_1')^2 = V_1^2 + v_{cm}^2 + 2V_1 v_{cm}\cos\Theta$	2.0 pts

5. Kinetic Energy of the neutron before collision $= E_1 = \frac{1}{2} m_1 v_1^2$

Kinetic Energy of the neutron after collision $= E_1' = \frac{1}{2} m_1 (v_1')^2$

From part (4)

$$(v_1')^2 = V_1^2 + v_{cm}^2 + 2V_1 v_{cm} \cos\Theta$$

As we know that $V_1 = v_1 - v_{cm}$

$$(v_1')^2 = (v_1 - v_{cm})^2 + v_{cm}^2 + 2(v_1 - v_{cm})v_{cm} \cos\Theta$$

$$(v_1')^2 = v_1^2 \left[\left(1 - \frac{v_{cm}}{v_1}\right)^2 + \frac{v_{cm}^2}{v_1^2} + 2\left(1 - \frac{v_{cm}}{v_1}\right)\left(\frac{v_{cm}}{v_1}\right) \cos\Theta\right]$$

Also we know that

$$v_{cm} = \frac{m_1 v_1}{m_1 + M_2}$$

$$\frac{v_{cm}}{v_1} = \frac{m_1}{m_1 + M_2} = \frac{1}{1 + \frac{M_2}{m_1}} = \frac{1}{1 + A}$$

So

$$(v_1')^2 = v_1^2 \left[\left(1 - \frac{1}{1 + A}\right)^2 + \left(\frac{1}{1 + A}\right)^2 + 2\left(1 - \frac{1}{1 + A}\right)\left(\frac{1}{1 + A}\right) \cos\Theta\right]$$

$$(v_1')^2 = v_1^2 \left[\frac{A^2 + 1 + 2A\cos\Theta}{(1 + A)^2}\right]$$

Kinetic Energy of the neutron after collision is given by

$$E_1' = \frac{1}{2} m_1 (v_1')^2 = \frac{1}{2} m_1 v_1^2 \left[\frac{A^2 + 2A\cos\Theta + 1}{(1 + A)^2}\right]$$

$$E_1' = E_1 \left[\frac{A^2 + 2A\cos\Theta + 1}{(1 + A)^2}\right]$$

$$\frac{E_1'}{E_1} = \left[\frac{A^2 + 2A\cos\Theta + 1}{(1 + A)^2}\right]$$

Maximum ratio $\frac{E_1'}{E_1}$ occurs at $\Theta = 0$; $\left.\frac{E_1'}{E_1}\right]_{\Theta=0} = \left[\frac{A^2+2A+1}{(1+A)^2}\right] = 1$

Minimum ratio $\frac{E_1'}{E_1}$ occurs at $\Theta = \pi$; $\left.\frac{E_1'}{E_1}\right]_{\Theta=\pi} = \left[\frac{A^2-2A+1}{(1+A)^2}\right] = \left[\frac{(A-1)^2}{(A+1)^2}\right]$

5. 3.0 pts

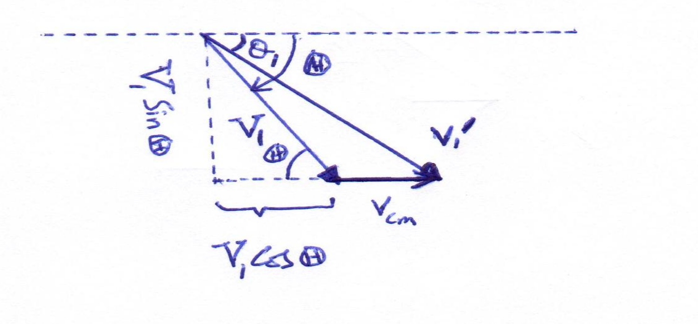
$$\frac{E_1'}{E_1} = \left[\frac{A^2 + 2A\cos\Theta + 1}{(1 + A)^2}\right]$$

Maximum ratio $\frac{E_1'}{E_1}$ occurs at $\Theta = 0$; $\left.\frac{E_1'}{E_1}\right]_{\Theta=0} = \left[\frac{A^2+2A+1}{(1+A)^2}\right] = 1$

Minimum ratio $\frac{E_1'}{E_1}$ occurs at $\Theta = \pi$; $\left.\frac{E_1'}{E_1}\right]_{\Theta=\pi} = \left[\frac{A^2-2A+1}{(1+A)^2}\right] = \left[\frac{(A-1)^2}{(A+1)^2}\right]$

6.

We need to find the energy of the scatted neutron in the laboratory frame of reference. From the figure in part (v) we can write



$$v_1' \cos\theta_1 = (v_1 - v_{cm})\cos\Theta + v_{cm}$$

$$v_1' \cos\theta_1 = \frac{A}{1+A} v_1 \cos\Theta + \frac{1}{1+A} v_1$$

$$\cos\theta_1 = \left[\frac{A}{1+A}\cos\Theta + \frac{1}{1+A}\right]\left(\frac{v_1}{v_1'}\right)$$

$$\cos\theta_1 = \left[\frac{A}{1+A}\cos\Theta + \frac{1}{1+A}\right]\left(\sqrt{\frac{E_1}{E_1'}}\right)$$

or $\cos\Theta = \left(\sqrt{\frac{E_1'}{E_1}}\cos\theta_1 - \frac{1}{1+A}\right)\left(\frac{1+A}{A}\right)$

Using it in equation $E_1' = E_1\left[\frac{A^2+2A\cos\Theta+1}{(1+A)^2}\right]$ after some simplifications one gets

$$E_1' = E_1\left[\frac{2\sqrt{\frac{E_1'}{E_1}}\cos\theta_1 + (A-1)}{(1+A)}\right]$$

$$(1+A)E_1' - 2\sqrt{E_1 E_1'}\cos\theta_1 - (A-1)E_1 = 0$$

$$\sqrt{E_1'} = \frac{\sqrt{E_1}\cos\theta_1 \pm \sqrt{E_1\cos^2\theta_1 + (A+1)(A-1)}}{(1+A)}$$

After simplifications

$$E_1' = \frac{E_1}{(1+A)^2}\left[\cos\theta_1 + \sqrt{\mathrm{A}^2 - \sin^2\theta_1}\right]^2$$

For $\theta_1 = 45^\circ$ *and* $A = 2$

$$E_1' = \frac{1\,MeV}{9}\left[\cos 45^\circ + \sqrt{4 - \sin^2 45^\circ}\right]^2 = 0.7384\,MeV$$

6.	$E_1' = 0.7384\,MeV$	3.0	pts

Q4. MASS OF A NEUTRON STAR (10 pts)

1.

$$m(Z,N)c^2 = Zm_Hc^2 + Nm_nc^2 - a_vA + a_sA^{\frac{2}{3}} + a_c\frac{Z^2}{A^{\frac{1}{3}}} + a_{sym}\frac{(N-Z)^2}{A}$$

Where $m_H = 938.8\frac{MeV}{c^2}$, $and\ m_n = 939.6\frac{MeV}{c^2}$,

1.	$m(Z,N)c^2 = Zm_Hc^2 + Nm_nc^2 - a_vA + a_sA^{\frac{2}{3}} + a_c\frac{Z^2}{A^{\frac{1}{3}}} + a_{sym}\frac{(N-Z)^2}{A}$	0.5 pts

2.

Mirror nuclei have the same value of A but interchanged values for N, Z. The only terms in the SEMF which survive when taking the mass difference between mirror nuclei are the Zm_Hc^2, Nm_nc^2 and $a_c\frac{Z^2}{A^{\frac{1}{3}}}$.

$$\Delta mc^2 = [m({}^{11}_{5}B) - m({}^{11}_{6}C)]c^2$$

$$= (Z_1 - Z_2)m_Hc^2 + (N_1 - N_2)m_nc^2 + a_c\frac{(Z_1^2 - Z_2^2)}{A^{\frac{1}{3}}}$$

Using given values of Z and masses gives $\Delta mc^2 = -2.66$ MeV
Difference shows ${}^{11}_{5}B$ more stable than ${}^{11}_{6}C$.
It means that ${}^{11}_{6}C$ will decay to ${}^{11}_{5}B$ by β^+ emission

2.	Using given values of Z and masses gives $\Delta mc^2 = -2.66$ MeV Difference shows ${}^{11}_{5}B$ more stable than ${}^{11}_{6}C$. It means that ${}^{11}_{6}C$ will decay to ${}^{11}_{5}B$ by β^+ emission	1.5 pts

3.

Under the condition of $Z = N$, the binding energy per nucleon is given by

$$\frac{B}{A} = a_v - a_sA^{-\frac{1}{3}} - \frac{a_c}{4}A^{\frac{2}{3}}$$

Where $Z = \frac{A}{2}$ has been used for the third term and the fourth term does not appear due to $Z = N$. As long as A is small, the second term is dominantly increasing with increasing A, and it is eventually taken over by the third term which is decreasing. Therefore, the extermal corresponds to the maximum of B/A. One can explicitly carry out

$$\frac{d(B/A)}{dA} = 0$$

to find the following condition,

$$\frac{a_s}{3}A^{-\frac{4}{3}} - \frac{a_c}{6}A^{-\frac{1}{3}} = 0$$

The solution is

$$A = \frac{2a_s}{a_c}$$

From the given numerical values, $A = 50$ (which must be an integer) is concluded.

Note: In reality B/A has a maximum for A ranging from ^{56}Fe to ^{62}Ni. The discrepancy from the answer in this problem is understood by the approximation of dropping the pairing energy and disregarding a mass difference between the proton and the neutron.

3.	$A = 50$	1.5 pts

4.

Take the differentiation of $(Z, A - Z)/A$ with respect to Z for a fixed A, which leads to

$$-2a_c \frac{Z^*}{A^{\frac{1}{3}}} - 4a_{sym} \frac{2Z^* - A}{A} = 0$$

By solving this in terms of Z^*, one finds

$$Z^* = \frac{1}{1 + \frac{a_c}{4a_{sym}} A^{\frac{2}{3}}} \frac{A}{2}$$

From this expression one can understand that $Z^* \simeq N$ as long as A is small enough, while Z^* becomes far smaller than N for large A. It is obvious from the explicit form that the symmetry energy tends to favor $Z = N$ but the Coulomb interaction tends to favor $Z \to 0$, and the balance between these competing effects determines $Z*$. Nuclei with too many neutrons (protons) would go through the $\beta-$ decay (the $\beta+$ decay or the electron capture) toward the stable (Z, N).

For A=197, $Z^* = 79$.

4.	For A=197, $Z^* = 79$	1.5 pts

5.

Fission process total nucleons remain the same. Terms dependent on A only will not contribute.

$$\Delta mc^2 = [m(^{238}_{92}\text{U}) - m(^{119}_{46}Pd)]c^2$$
$$= Zm_Hc^2 + Nm_nc^2 - \frac{B(^{238}_{92}\text{U})}{c^2}$$
$$- 2\left(Z_1m_Hc^2 + N_1m_nc^2 - \frac{B(^{119}_{46}\text{Pd})}{c^2}\right)$$

In symmetric fission Z_1=Z/2 and N_1=N/2. Therefore using the SEMF.

$$\Delta mc^2 = 2\frac{B(^{119}_{46}\text{Pd})}{c^2} - \frac{B(^{238}_{92}\text{U})}{c^2}$$
$$= 2(1856.4 - 416.13 - 301.13 - 142.74 + 0) - (3712.8 - 660.56 - 956.05 - 285.47 + 0.77)$$
$$= 181 MeV$$

5.	Fission products are often unstable as you have gone from a heavy nucleus where the most stable configuration favours a large neutron excess, to a lighter nucleus where the most stable configuration has smaller (or zero) neutron excess. Therefore the fission fragments tend to have an excess of neutrons compared to the stable configuration and are unstable	1.5 pts

6.

The expression for the binding energy per

$$\frac{B}{A} = a_v - a_s A^{-\frac{1}{3}} - a_c \frac{Z^2}{A^{\frac{4}{3}}} - a_{sym} \frac{(A-2Z)^2}{A^2}$$

As A is very large the surface and coulomb terms are negligible. The B/A is determined by the difference between the volume term and the asymmetry term. The latter approaches if we assume the star is dominantly neutrons i.e. A>>Z

$$\frac{B}{A} = a_v - a_{sym} = 15.6 - 23.3 = -7.7\ MeV$$

The expression is negative implying the system is unbound. Therefore to bind a star other forces contribute.

6.	The expression is negative implying the system is unbound. Therefore to bind a star other forces contribute.	1.0 pts

7.

The expression apart from the parametric dependence on A can be identified as

$$a_{grav} = \frac{3Gm_n^2}{5R_0}$$

which is re-expressed in terms of M_P using the given relation to G, leading to

$$a_{grav} = \frac{3\hbar c\ m_n^2}{5R_0 M_P^2}$$

$$a_{grav} = \frac{3}{5}\ \frac{197\ fm \cdot MeV \times (939\ MeV/c^2)^2}{1.1\ fm \times (1.22 \times 1022\ MeV/c^2)^2} \approx 6.4 \times 10^{-37}\ MeV$$

The stability is judged from the condition that the binding energy should be positive, i.e.

$$B(A) = a_v A - a_{sym} A + a_{grav} A^{\frac{5}{3}} > 0$$

This inequality can be translated into $A > A_c$ with A_c given by

$$A_c = \left(\frac{a_v - a_{sym}}{a_{grav}}\right)^{3/2} \cong 4.4 \times 10^{55}$$

7.	4.4×10^{55}	2.5 pts

Q5. RADIOACTIVE DATING (10 pts)

Part 1. Naturally Occurring Uranium

1.1

Suppose that we have one gram of natural Uranium

Amount of ${}^{235}_{92}U$ in one gram of natural uranium=0.007 gram

Amount of ${}^{238}_{92}U$ in one gram of natural uranium=0.993 gram

$$\text{No of Atoms of } {}^{235}_{92}U \ : \ \text{No of Atoms of } {}^{238}_{92}U$$

$$\frac{0.007}{235}\mathrm{N_A} : \frac{0.993}{238}\mathrm{N_A}$$

$$1 : 140$$

There are 140 atoms of ${}^{238}_{92}U$ for each atom of ${}^{235}_{92}U$ in natural Uranium

1.1	There are 140 atoms of ${}^{238}_{92}U$ for each atom of ${}^{235}_{92}U$	1.0 pts

1.2

$$n^{235}(t) = n_0^{235} e^{-\lambda^{235} t}$$
$$n^{238}(t) = n_0^{238} e^{-\lambda^{238} t}$$
$$\lambda^{235} = \frac{\ln 2}{T_{\frac{1}{2}}^{235}} = 9.72 \times 10^{-10}\ year^{-1}$$
$$\lambda^{238} = \frac{\ln 2}{T_{\frac{1}{2}}^{238}} = 1.54 \times 10^{-10}\ year^{-1}$$
$$\frac{n^{238}(t)}{n^{235}(t)} = \frac{n_0^{238}}{n_0^{235}} e^{(\lambda^{235} - \lambda^{238})t}$$

It is given that initially $n_0^{238} = n_0^{235}$

$$140 = e^{(\lambda^{235} - \lambda^{238})t}$$
$$t = \frac{1}{(\lambda^{235} - \lambda^{238})}\ln(140)$$

$$t = 6.04 \times 10^9 \text{ years}$$

1.2	6.04×10^9 years	2.0 pts

1.3

$$T_{1/2}^{235} = 7.13 \times 10^8 \; years$$

$$\lambda^{235} = \frac{\ln 2}{T_{\frac{1}{2}}^{235}} = 9.72 \times 10^{-10} \; year^{-1}$$

$$T_{1/2}^{238} = 4.51 \times 10^9 \; years$$

$$\lambda^{238} = \frac{\ln 2}{T_{\frac{1}{2}}^{238}} = 1.54 \times 10^{-10} \; year^{-1}$$

$$n^{235}(t) = n_0^{235} e^{-\lambda^{235} t}$$

$$n^{238}(t) = n_0^{238} e^{-\lambda^{238} t}$$

At $t = 0$, isotopic abundance of $U^{235} = 0.03 = \gamma_{235}$

Since n^{235} and $n^{238} \gg n^{234}$

$$\gamma_{235} = \frac{n_0^{235}}{n_0^{235} + n_0^{234} + n_0^{238}} = \frac{n_0^{235}}{n_0^{235} + n_0^{238}} = 0.03$$

$$n_0^{235} = 0.03 n_0^{235} + 0.03 n_0^{238}$$

$$0.97 n_0^{235} = 0.03 n_0^{238}$$

$\frac{n_0^{235}}{n_0^{238}} = \frac{0.03}{0.97} = 0.03093$ at $t = 0$

Presently $\gamma_{235} = 0.0072 = \frac{n^{235}}{n^{235}+n^{238}}$

$$0.9928 n^{235} = 0.0072 n^{238}$$

$$\frac{n^{238}}{n^{235}} = \frac{0.9928}{0.0072} = \frac{1}{0.00725}$$

Dividing the two decay equations

$$\frac{n^{235}(t)}{n^{238}(t)} = \frac{n_0^{235}}{n_0^{238}} e^{(\lambda^{238} - \lambda^{235})t}$$

$$t = \frac{1}{(\lambda^{238} - \lambda^{235})} \ln\left(\frac{n^{235}(t)/n^{238}(t)}{n_0^{235}/n_0^{238}}\right)$$

$$t = \frac{1}{(1.54 \times 10^{-10} - 9.72 \times 10^{-10})} \ln\left(\frac{0.00725}{0.03093}\right)$$

$$t = 1.77 \times 10^9 \; years$$

1.3	$1.77 \times 10^9 \; years$	3.0 pts

Part 2. Radioactive tracer

2.1

$N = N_0 e^{-\lambda t}$, where $\lambda = \ln 2/45$ days^{-1}

$$N(30) = 4 \times 10^5\ e^{-30\lambda} = 4 \times 10^5\ e^{-\frac{30}{45}\ln 2} = 2.52 \times 10^5\ Bq$$

Fraction of the mass removed:

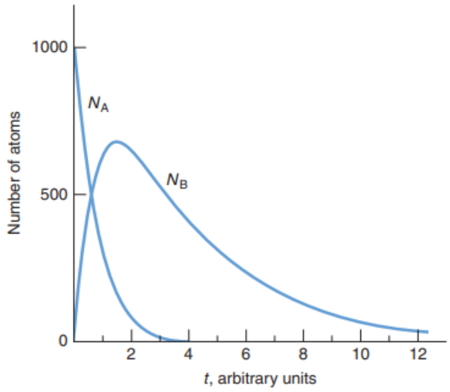
Activity in $5 \times 10^{-3}\ m^3$ of oil$= \frac{126}{10\times 60}\left(\frac{5\times 10^{-3}}{10^2\times 10^{-6}}\right) = \frac{126}{600}(50) = 10.5\ Bq$

$$\text{Fraction of mass removed} = \frac{10.5}{2.52\times 10^5} = 4.2 \times 10^{-5}$$

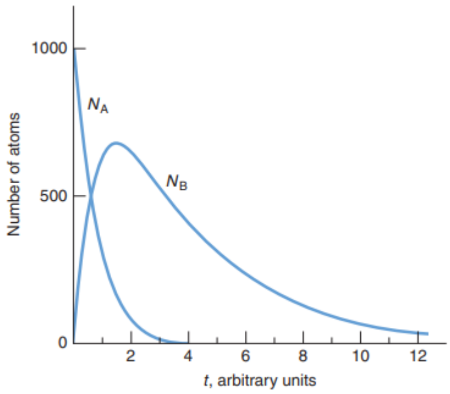
2.1	4.2×10^{-3} %	4.0	pts

Q6. RADIOACTIVE EQUILIBRIUM (10 pts)

1.



1. 1.0 pts



2.

$$R = \frac{\lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

For small values of $\boldsymbol{t}$**,** $e^{-\lambda_1 t} = 1 - \lambda_1 t + \cdots \quad and \;\; e^{-\lambda_2 t} = 1 - \lambda_2 t + \cdots$

$$R = \frac{\lambda_1}{\lambda_2 - \lambda_1}[1 - \lambda_1 t + \cdots - (1 - \lambda_2 t + \cdots)]$$

$$R = \frac{\lambda_1}{\lambda_2 - \lambda_1}[(\lambda_2 - \lambda_1)t + \cdots] \approx \lambda_1 t$$

For large values of $\boldsymbol{t}$**,** and $\lambda_2 \gg \lambda_1$, the second exponential term can be neglected, then

$$R = \frac{\lambda_1}{\lambda_2 - \lambda_1}e^{-\lambda_1 t} \approx \frac{\lambda_1}{\lambda_2}e^{-\lambda_1 t}$$

2. 2.0 pts

$$R \approx \lambda_1 t$$

$$R \approx \frac{\lambda_1}{\lambda_2}e^{-\lambda_1 t}$$

3.

For small values of t, $R \approx \lambda_1 t$

Plot R against t for small t, (in the region where $e^{-\lambda t} = 1 - \lambda t$). This will be a straight line with slope λ_1. Hence λ_1 can be measured by the slope of the straight line.

For large values of t,

$$R = \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_1 t}$$

By taking log on both sides

$$\ln R = \ln \frac{\lambda_1}{\lambda_2 - \lambda_1} - \lambda_1 t$$

Plot $\ln R$ against t for large values t, (in the region where $e^{-\lambda_1 t} \gg e^{-\lambda_2 t}$). This will be a straight line with slope λ_1 and intercept $\ln \frac{\lambda_1}{\lambda_2 - \lambda_1}$. λ_1 has already been determined, λ_2 can be determined from the intercept as follows

$$k = \ln \frac{\lambda_1}{\lambda_2 - \lambda_1}$$

By taking anti-log on both sides

$$\frac{\lambda_1}{\lambda_2 - \lambda_1} = e^k$$

$$\lambda_2 = \lambda_1 (1 + e^{-k})$$

3.	As described above	2.0 pts

4.

As we have

$$R = \frac{N_2(t)}{N_0} = \frac{\lambda_1}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

Also

$$N_1(t) = N_0 e^{-\lambda_1 t}$$

For $\lambda_2 \gg \lambda_1$ and $e^{-\lambda_1 t} \gg e^{-\lambda_2 t}$ (after suitable long times) we get

$$\frac{N_2}{N_1} = \frac{\lambda_1}{\lambda_2 - \lambda_1} = \frac{\lambda_1}{\lambda_2}$$

This implies that

$$N_2 \lambda_2 = N_1 \lambda_1$$

Conditions for validity are $\lambda_2 \gg \lambda_1$ and $e^{-\lambda_1 t} \gg e^{-\lambda_2 t}$.

4.	Conditions for validity are $\lambda_2 \gg \lambda_1$ and $e^{-\lambda_1 t} \gg e^{-\lambda_2 t}$	3.0 pts

5.

$$N_3(t) = N_0 - N_1(t) - N_2(t)$$

$$N_3(t) = N_0 - N_0 e^{-\lambda_1 t} - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

$$N_3(t) = N_0 - \frac{\left(\lambda_2 N_0 e^{-\lambda_1 t} - \lambda_1 N_0 e^{-\lambda_1 t} + \lambda_1 N_0 e^{-\lambda_1 t} - \lambda_1 N_0 e^{-\lambda_2 t}\right)}{\lambda_2 - \lambda_1}$$

$$N_3(t) = N_0 - N_0 \frac{\left(\lambda_2 e^{-\lambda_1 t} - \lambda_1 e^{-\lambda_2 t}\right)}{\lambda_2 - \lambda_1}$$

5.	$N_3(t) = N_0 - N_0 \frac{\left(\lambda_2 e^{-\lambda_1 t} - \lambda_1 e^{-\lambda_2 t}\right)}{\lambda_2 - \lambda_1}$	2.0 pts

Q7. RADIOACTIVITY AS SOURCE OF HEAT (10 pts)

1. Total energy is equal to.

$$Q = [m({}^{210}_{84}Po) - m({}^{206}_{82}Pb) - m({}^{4}_{2}He)]c^2$$
$$Q = [209.98287 - 205.97447 - 4.0026] \times 1.6605 \times 10^{-27} \times (2 \times 10^8)^2$$
$$Q = 8.66801 \times 10^{-13}\, J = 5.403\, MeV$$

1.	$Q = 8.66801 \times 10^{-13}\, J = 5.403\, MeV$	1.0 pts

2.

Conservation of momentum gives

$$m_\alpha v_\alpha - m_D v_D = 0$$

This equation gives

$$v_\alpha = \frac{m_D v_D}{m_\alpha}$$

Kinetic Energy of the Daughter Nucleus i.e., $({}^{206}_{82}Pb) = K_D = \frac{1}{2} m_D v_D^2$

Kinetic Energy of the Alpha Particle $= K_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 = \frac{1}{2} m_\alpha \left(\frac{m_D v_D}{m_\alpha}\right)^2 = \frac{m_D}{m_\alpha} K_D$

Q value= Total Kinetic Energy$= K_\alpha + K_D = K_\alpha + \frac{m_\alpha}{m_D} K_\alpha = K_\alpha \left(1 + \frac{m_\alpha}{m_D}\right)$

$$K_\alpha = \frac{Q}{\left(1 + \frac{m_\alpha}{m_D}\right)}$$

By putting the corresponding values one gets

$$K_\alpha = \frac{5.403}{\left(1 + \frac{4.0026}{205.97447}\right)} = 5.3\, MeV$$

The kinetic energy of the daughter is given by

$$K_D = Q - K_\alpha = 0.103\, MeV$$

2.	$K_\alpha = 5.3\, MeV$	3.0 pts

3.

The average/mean life time$= \tau = \frac{1}{\lambda}$ where λ is decay constant.

Number of nuclei decayed during time $t = N = N_0 - N_0 e^{-\lambda t}$

Where $N_0 = \frac{M}{209.98287} N_A = \left(\frac{1.0\times10^{-6}}{209.98287}\right) \times 6.022 \times 10^{23} = 2.8678529 \times 10^{15}\ Atoms\ (Nuclei)$ is the initial number of nuclei of ${}^{210}_{84}Po$.

$N_A = 6.022 \times 10^{23}$ is the Avogadro's number.

Number of nuclei decayed during average life time $N = N_0(1 - e^{-1}) = 2.8678529 \times 10^{15} \times 0.632109 = 1.812797 \times 10^{15}$ Decays.

Total energy released due to decay process during average life time $= QN$

$$QN = 5.403 \times 1.812797 \times 10^{15} = 9.79454 \times 10^{15} MeV$$
$$= 9.79454 \times 10^{15} \times 1.602 \times 10^{-13}\ J = 1.569\ KJ$$

3.	$1.569\ KJ$	3.0 pts

4.

Energy released per unit activity$= \frac{5.3\times10^{6}\ eV}{disintegration} \times \frac{3.7\times10^{10}\ disintegrations/s}{1\ Ci} \times \frac{1.602\times10^{-19}\ J}{1\ eV} = 0.0314056 \frac{W}{Ci}$

Activity required to generate $0.0314056\ W$ Energy = 1 Ci
Activity required to generate $1.0W$ Energy $= \frac{1}{0.0314056}\ Ci$
Activity required to generate $10^6\ W$ Energy $= \frac{10^6}{0.0314056}\ Ci = 3.184 \times 10^7\ Ci = 3.184 \times 10^7\ Ci \times \frac{3.7\times10^{10}\ disintegrations/s}{1\ Ci} = 1.17808 \times 10^{18}\ disintegrations/sec$

$$Activity = \lambda N$$

$$N = \frac{Activity}{\lambda}$$

$$\lambda = \frac{0.693}{138 \times 24 \times 3600} = 5.8134 \times 10^{-8}\ \ sec^{-1}$$

$$N = \frac{1.17808 \times 10^{18}}{5.8134 \times 10^{-8}} = 2.02649 \times 10^{25}\ disintergrations$$

Mass of corresponding ${}^{210}_{84}Po = \left(\frac{2.02649\times10^{25}}{6.022\times10^{23}}\right) \times 209.98287 = 7.066 \times 10^{3}\ gram = 7.066\ Kg$

4.	$7.066\ Kg$	3.0 pts

Q8. MODEL OF THE ATOM (10 pts)

Part 1. Plum pudding model of the atom (4.2 pts)

1.1 Since we are considering the "plum pudding" model for hydrogen atom, $Z=1$.

Therefore, nuclear charge dens ity of the **positively charged** cloud $= \frac{e}{\frac{4}{3}\pi a^3} = \frac{3e}{4\pi a^3}$

1.1	$\frac{3e}{4\pi a^3}$	0.4 pts

1.2 Here we are dealing with the characteristics inside the charged cloud and therefore must concentrate in the region where $r < a$.

Electric field at a distance r from the center $= \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{4}{3}\pi r^3\right)\left(\frac{3e}{4\pi a^3}\right)}{r^2} = \frac{e}{4\pi\varepsilon_0 a^3}r$ Note that this electric field is in the radially outward direction.

1.2	$\frac{e}{4\pi\varepsilon_0 a^3}r$	0.8 pts

1.3 In this case, force acting on the electron is always directed towards the center.

Therefore, $m\frac{d^2r}{dt^2} = -\frac{e^2}{4\pi\varepsilon_0 a^3}r$ which leads to $\frac{d^2r}{dt^2} = -\omega^2 r$ with $\omega = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m a^3}}$

1.3	For the equation $m\frac{d^2r}{dt^2} = -\frac{e^2}{4\pi\varepsilon_0 a^3}r$	0.6 pts
	$\omega = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m a^3}}$	0.4 pts

1.4 Because the Simple Harmonic Motion (SHM) described by the equations in (iii) above is restricted to the inside of the charge cloud, the amplitude of the SHM can be considered as a.

$$\therefore \text{Maximum speed} = Amplitude\ of\ the\ motion \times \omega \ = e\sqrt{\frac{1}{4\pi\varepsilon_0 ma}}$$

Substituting $a \sim 1$ fm $= 10^{-15}$ m, $\varepsilon_0 = 8.85 \times 10^{-12}$ F. m^{-1}, $m = 9.11 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C.

One gets maximum speed $= 4.8 \times 10^8$ m s^{-1}. From the theory of SHM, this maximum speed occurs at the center of the charge cloud.

1.4	Maximum speed $= 4.8 \times 10^8$ m s^{-1}	1.4 pts

1.5

1.5	The value we get for the maximum speed of a negative charge is greater than the speed of light in a vacuum and therefore the model cannot be correct.	0.6 pts

Part 2. Coulomb Barrier (5.8 pts)

2.1 Assuming a spherical shape, calculate the radius of the Fe nuclide.

Here, the minimum kinetic energy required to overcome the Coulomb barrier of the nucleus must be equal to electromagnetic potential energy of a proton at the surface of the nucleus. $\therefore KE_{min} = \frac{Ze^2}{4\pi\varepsilon_0 R}$

$$\therefore R = \frac{Ze^2}{4\pi\varepsilon_0(KE_{min})} = \frac{26\times(1.602\times10^{-19})^2}{4\times\pi\times8.85\times10^{-12}\times1.42\times10^{-12}} = 4.23 \text{ fm}$$

2.1	4.23 fm	1.4 pts

2.2 Radius of the ^{4}He is given as 1.74 fm. Use this information to find the mass number of Fe isotope used as the target.

Use the equation $R = r_0 A^{1/3}$. $\therefore \left(\frac{4.23}{1.74}\right) = \left(\frac{A}{4}\right)^{\frac{1}{3}}$ $A = 57$

[Note: Depending on the way one does the numerical calculation, he/she might get the answer as 58, which can also be considered as correct. If A is taken as 58, then the answers to following (iii), (iv) and (v) must be changed accordingly]

2.2	$A = 57$	1.0 pts

2.3 Total B.E.

$$= [26 \times 1.672622 + 31 \times 1.674928 - 94.51982] \times 10^{-27} \times (3 \times 10^8)^2 \text{J}$$
$$= 8.02 \times 10^{-11} \text{ J}$$

Binding Energy per nucleon $= 1.41 \times 10^{-12} \frac{\text{joules}}{\text{nucleon}} = 8.8 \frac{\text{MeV}}{\text{nucleon}}$

2.3	$8.8 \frac{\text{MeV}}{\text{nucleon}}$	1.2 pts

2.4 Density of nuclear matter= $\rho_N = \frac{9.451982 \times 10^{-26}}{\frac{4}{3}\pi(4.23 \times 10^{-15})^3} \sim 3 \times 10^{17} \text{ kg.m}^{-3}$

Densities as high as this can be found in neutron stars.

2.4	$\rho_N \sim 3 \times 10^{17} \text{ kg.m}^{-3}$ Neutron stars (or other acceptable answer)	1.0 pts 0.2 pts

2.5

2.5	Positively charged quarks = 83	0.5 pts
	Negatively charged quarks = 88	0.5 pts

Q9. H3-DECAY (10 pts)

Part 1. Radioactive decay (6.5 pts)

1.1

1.1	p$_c$: β^{-1} (Beta particle or electron)	0.25 pts
	p$_n$: $\bar{\nu}_e$ (Anti electron-neutrino)	0.25 pts

1.2

1.2	p$_c$ (or β^{-1}) can be detected easily	0.6 pts
	Because p$_n$ (or $\bar{\nu}_e$) is uncharged particle with almost zero mass.	0.4 pts

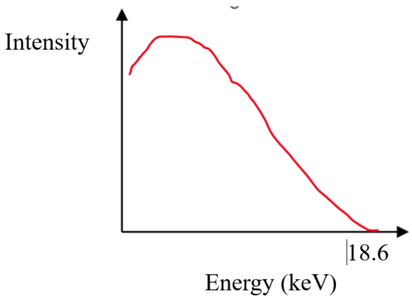
1.3

Maximum kinetic energy = $[m(^3\text{H}) - m(^3\text{He})]c^2 = 18.6$ keV

1.3	18.6 keV	1.0 pts

1.4

1.4



For the shape of the curve	0.8 pts
For indicating the maximum value on the energy axis	0.2 pts

1.5

1.5	The energy spectrum for alpha particles show discrete peaks as opposed to a continuous spectrum similar to the one depicted in (iv) above.	1.0 pts

1.6

1.5	Energy of p_n = 18.6-16.1 = 2.5 keV	2.0 pts

Part 2. Enriched hydrogen (3.5 pts)

2.1

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{12.33 \times 365.25 \times 24 \times 60 \times 60} = 1.78 \times 10^{-9}\ \mathrm{s}^{-1}$$

2.1	$\lambda = 1.78 \times 10^{-9}\ \mathrm{s}^{-1}$ (or equivalent value in yr^{-1})	0.5 pts

2.2

To find the initial activity, one must know the number of tritium atoms that exist in the sample. From the nuclear data sheets, mass of a tritium atom is 3.0160495 u or 5×10^{-24} g.

Therefore number of tritium atoms in a 0.1 g sample $= \frac{0.1}{5 \times 10^{-24}} \sim 2 \times 10^{22}$

[Note that you get the same answer by considering the molar mass of tritium which is 3.016 g.]

$$\therefore \text{Initial activity} = 2 \times 10^{22} \times 1.78 \times 10^{-9}\ \mathrm{s}^{-1} = 3.56 \times 10^{13}\ \mathrm{Bq}$$

2.2	3.56×10^{13} Bq	1.0 pts

2.3

$$\text{Average energy} = \frac{88}{60 \times 60 \times 3.56 \times 10^{13}}\ \mathrm{J} = 6.87 \times 10^{-16}\ \mathrm{J} = 4.3\ \mathrm{keV}$$

2.3	4.3 keV	2.0 pts

Q10 RADIOACTIVE POWER SYSTEMS (10 pts)

Part 1. Reaction Energy and Half-life Calculation (5.8 pts)

1.1

Emission of an alpha particle from $^{238}_{94}\text{Pu}$.

Emission of a deuteron from $^{238}_{94}\text{Pu}$.

Two processes are,

$$^{238}_{94}\text{Pu} \rightarrow {}^{234}_{92}\text{U} + \alpha$$
$$Q = (238.049553 - 234.040950 - 4.001506)931.5 \text{ MeV}$$
$$Q = 6.611 \text{ MeV}$$

$$^{238}_{94}\text{Pu} \rightarrow {}^{236}_{93}\text{Np} + \text{d}$$
$$Q = (238.049553 - 236.046570 - 2.013553)931.5 \text{ MeV}$$
$$Q = -9.846 \text{ MeV}$$

1.1	$Q = 6.611$ MeV $Q = -9.846$ MeV	1.0 pts 1.0 pts

1.2

1.2	Based on the change in energy values, the second process is not spontaneous.	0.8 pts

1.3

Total K.E. emitted per decay = $E_t = E_\alpha + E_U = \frac{{p_\alpha}^2}{2m_\alpha} + \frac{{p_\alpha}^2}{2m_U} = E_\alpha\left(1 + \frac{m_\alpha}{m_U}\right)$

$$= 5.5\left(\frac{238}{234}\right) = 5.6 \text{ MeV}$$

[Note that $p_\alpha = p_U$, from the conservation of momentum]

1.3	5.6 MeV	1.4 pts

1.4

Number of atoms in 120 g of ^{238}Pu = N

$$= \frac{120}{1.66054\times10^{-24}\times238.049553} = 3.04\times10^{23}$$

$$\text{Decay rate} = \left(\frac{dN}{dt}\right) = \frac{68.2}{5.6\times1.60218\times10^{-13}} = 7.60\times10^{13}\ decays.\text{s}^{-1}$$

$$\text{Decay constant} = \lambda = \frac{\left(\frac{dN}{dt}\right)}{N} = \frac{7.60125\times10^{13}}{3.03574\times10^{23}} = 2.50\times10^{-10}\ s^{-1}$$

$$\therefore Half\ life = t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{2.50392\times10^{-10}}\ \text{s} = 2.77\times10^{9}\ \text{s} = 87.7\ \text{years}$$

1.4		
	Decay constant = $\lambda = 2.50\times10^{-10}\ s^{-1}$	0.6 pts
	Half life $= 2.77\times10^{9}$ s or 87.7 years	1.0 pts

Part 2. Given mass of Plutonium (4.2 pts)

2.1

$$\text{Decay constant } \lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{87.7\times365.25\times24\times60\times60} = 2.5\times10^{-10}\ s^{-1}$$

$$\text{Initial Power} = \frac{dE}{dt} = E_t\left(\frac{dN}{dt}\right) = E_t\lambda N_0 = 5.6\times2.5\times10^{-10}\times2\times6.02\times10^{23}$$

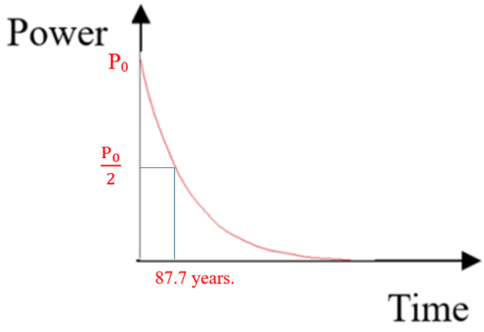
$$= 1.69\times10^{15}\ \text{MeV.s}^{-1}$$

[Note that number of moles of $^{238}_{94}$Pu in 476 g is 2]

2.1	$Initial\ Power = 1.69\times10^{15}\ \text{MeV}.s^{-1}$	1.2 pts

2.2

Power variation is same as the variation of activity because,
$Power = \frac{dE}{dt} = E_t\left(\frac{dN}{dt}\right) = E_t A(t)$ with E_t constant. Therefore power becomes half during a period equal to the half-life which is 87.7 years.

2.2		
	For the shape of the curve	0.4 pts
	Indicating power becomes half during a period of 87.7 years	0.2 pts

2.3

Because the initial power is sixteen times more than the minimum power required, the RPS would generate less than the minimum power after four half-lives.
Therefore, lifetime of RPS = $87.7 \times 4 = 350\ years$!!!!!!!!!!!!!

2.3	Lifetime of RPS = $87.7 \times 4 = 350\ years$	1.2 pts

2.4

Lifetime of the RPS = $4t_{\frac{1}{2}}$

∴Activity at the end of the lifetime of the RPS = $\frac{Initial\ Activity}{16}$

$$= \frac{2.5\times10^{-10}\times2\times6.02\times10^{23}}{16} = 1.88\times10^{13}\ \text{Bq}$$

2.4	1.88×10^{13} Bq	1.2 pts

Q11 RADIOISOTOPE APPLICATION (10 pts)

Part 1. (3.4 pts)

1.1

$$\text{Total decays per second} = 3.7 \times 10^4$$

$$\text{Total number of } \gamma \text{ rays emitted per second} = \left(\frac{85.1}{100}\right) \times 3.7 \times 10^4$$

$$= 3.15 \times 10^4$$

1.1	3.15×10^4	1.2 pts

1.2

$$\text{Absorbed dose} = \frac{3.15\times10^4\times0.662\times1.602\times10^{-13}\times2\times60\times60\times.05}{75}\ \text{Gy}$$

$$= 1.6 \times 10^{-8}\ \text{Gy} = 1.6 \times 10^{-6}\ \text{rad}$$

Dose Equivalent = Absorbed dose ×Quality factor

The currently accepted value of quality factor for gamma radiation is 1.

Dose Equivalent = 1.6×10^{-8} Sv = 1.6×10^{-6} rem

1.2	Absorbed dose = 1.6×10^{-8} Gy = 1.6×10^{-6} rad	0.7 pts
	Dose Equivalent = 1.6×10^{-8} Sv = 1.6×10^{-6} rem	0.7 pts

1.3

Absorbed dose = 1.6×10^{-8} Gy = 1.6×10^{-6} rad

The currently accepted value of quality factor for alpha particles is 20.

Therefore, Dose Equivalent = 3.2×10^{-7} Sv = 3.2×10^{-5} rem

1.3	Absorbed dose = 1.6×10^{-8} Gy = 1.6×10^{-6} rad	0.4 pts

Dose Equivalent = 3.2×10^{-7} Sv = 3.2×10^{-5} rem	0.4 pts

Part 2. (4.2 pts)

2.1

Minimum count rate is expected when the detector and the source are at opposite corners of the room. i.e. when the distance between the source and the detector is $\sqrt{(288)}$ m.

$$\gamma\ ray\ flux\ density\ at\ the\ position\ of\ the\ detector = \frac{3.15\times10^4}{4\pi\times288}\ \mathrm{s^{-1}m^{-2}}$$

$$Minimum\ count\ rate = \frac{3.15\times10^4}{4\pi\times288} \times 10^{-3} \times 0.8 \text{ counts per second}$$

$$= 0.007\ \mathrm{s^{-1}}$$

This is a very low count rate and cannot be observed due to the typical background level.

2.1	Minimum Count Rate = 0.007 s^{-1}	1.5 pts
	Very small count rate compared to typical background level	0.5 pts

2.2

If this distance is d, $\frac{3.15\times10^4\times10^{-3}\times0.8}{4\pi d^2} = 20$, and therefore $d \approx 32$ cm.

2.2	$d \approx 32$ cm.	1.6 pts

2.3

2.3	This particular radiation detector cannot be used to locate the source because the count rates are very low in general and detector must be at a distance closer than 32 cm to have a measurable count rate.	0.6 pts

Part 3. (2.4 pts)

3.1

Number of decays required at the detector, to have a count rate of 10 s^{-1} = 40 decays s^{-1}
(Because the overall efficiency is 25%.)
We must consider the lower range of speed for the measurement to be successful.
Time between the addition of source and detection = $\frac{500}{0.4}\ s = 1.25\times 10^3\ s$

$$\therefore\ 40 = A(0)e^{-\frac{0.693\times 1.25\times 1000}{32\times 60}}$$

Required activity for the added tracer = $A(0)$ = 62.8 Bq

3.1	For the correct application of the equation $40 = A(0)e^{-\frac{0.693\times 1.25\times 1000}{32\times 60}}$	1.4 pts
	Required activity for the added tracer = $A(0)$ = 62.8 Bq	0.5 pts

Q12 NUCLEAR BINDING ENERGY (10 pts)

Part 1. Nuclear Binding Energy (2.5 pts)

1.1

1.1	By definition $B = \left[Zm_p + Nm_n - mass\ of\ the\ nucleus\right]c^2$ Here $m_p = mass\ of\ the\ proton,$ $m_n = mass\ of\ the\ neutron$ and $N = A - Z$.	1.0 pts

1.2

1.2	$B = \left[Zm_p + Nm_n - (m(^A\text{X}) - Zm_e)\right]c^2$ $B = \left[Z(m_p + m_e) + Nm_n - m(^A\text{X})\right]c^2$ $B = [Zm(^1\text{H}) + Nm_n - m(^A\text{X})]c^2$	0.8 pts 0.7 pts

Part 2. (5.0 pts)

2.1

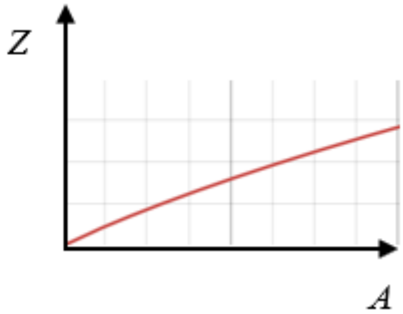
For the most stable nucleus $\frac{\partial B(Z,A)}{\partial Z} = 0$, which gives $Z = \frac{A}{2+\frac{a_3}{2a_4}A^{2/3}} = \frac{A}{2+0.0154A^{2/3}}$

2.1	For applying the condition $\frac{\partial B(Z,A)}{\partial Z} = 0$	0.6 pts
	For derivation in the form $Z = \frac{A}{2+\frac{a_3}{2a_4}A^{2/3}}$	0.8 pts
	For the derivation final expression $Z = \frac{A}{2+0.0154A^{2/3}}$	0.6 pts

2.2

$Z \sim \frac{A}{2}$ for smaller values of A and therefore gradient of the graph$\sim \frac{1}{2}$.

For larger values of A, gradient $< \frac{1}{2}$.

2.2	 For the correct shape of the curve For smaller values of A, gradient of the graph~1/2	0.4 pts 0.1 pts

2.3

2.3	If A =125, then Z = 52.4 and therefore most stable isotope is $_{52}$Te.	1.0 pts

2.4

Neutron separation energy (normally represented by the symbol S_n) for a nucleus ${}^{A}_{Z}X_N$ is the energy required to remove one neutron from the nucleus. Thus, S_n is equal to the difference in binding energies between ${}^{A}_{Z}X_N$ and ${}^{A-1}_{Z}X_{N-1}$.

Therefore $S_n = B({}^{A}_{Z}X_N) - B({}^{A-1}_{Z}X_{N-1})$.

For ${}^{125}_{52}\mathrm{Te}_{73}$, $S_n = B\left({}^{125}_{52}\mathrm{Te}_{73}\right) - B({}^{124}_{52}\mathrm{Te}_{72}) = B(52{,}125) - B(52{,}124)$

Substituting numerical values $B(52{,}125) = 1049.52$ MeV and $B(52{,}124) = 1042.24$ MeV. Therefore, $S_n = 1049.52 - 1042.24 = 7.28$ MeV

2.4	$B(52{,}125) = 1049.52$ MeV	0.5 pts
	$B(52{,}124) = 1042.24$ MeV	0.5 pts
	$S_n = 7.28$ MeV	0.5 pts

Part 3. (2.5 pts)

3.1

Calculate $B(20,39)$ and $B(19,39)$ from the given expression for $B(Z,A)$, and hence calculate $\Delta B = B(19,39) - B(20,39)$. Value of $\Delta B = 8.21$ MeV

[Numerical values for the 1st, 2nd and 4th terms are the same for both $B(19,39)$ and $B(20,39)$. Therefore, you simply have to subtract only the 3rd terms to get ΔB.]

Equating ΔB and ΔE_c,

$$\Delta E_c = \frac{3}{20\pi\varepsilon_0 R}\{(20e)^2 - (19e)^2\} = \frac{117e^2}{20\pi\varepsilon_0 R_0 (39)^{1/3}} = 8.21 \times 1.602 \times 10^{-13} \text{ J}$$

Substituting numerical values for e and ε_0 in the above expression one can find R_0 as R_0

3.1	$\Delta B = 8.21$ MeV	1.0 pts
	$\Delta E_c = \frac{3}{20\pi\varepsilon_0 R}\{(20e)^2 - (19e)^2\} = \frac{117e^2}{20\pi\varepsilon_0 R_0 (39)^{1/3}}$	1.0 pts
	$R_0 = 1.2$ fm	0.5 pts

Q13 RADIATION DETECTION (10 pts)

Part 1. Alpha particle in a magnetic field (6.5 pts)

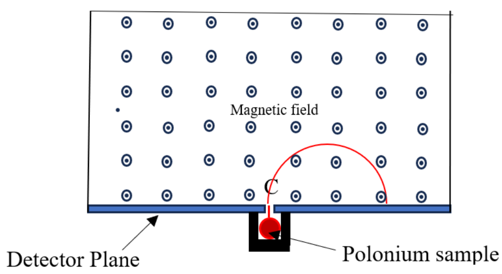
1.1

$$^{214}_{82}\text{Po} \rightarrow {}^{210}_{80}\text{Pb} + {}^{4}_{2}\text{He}$$

1.1	$^{214}_{82}\text{Po} \rightarrow {}^{210}_{80}\text{Pb} + {}^{4}_{2}\text{He}$	0.5 pts

1.2

1.2



For the correct (semicircular to the right side) path as indicated — 0.5 pts

1.3

At the minimum energy of not hitting the detector plane, radius of the chamber = twice the radius of the path.

By equating the magnitude of the centrifugal force to the force due to magnetic field,

$$\frac{m_\alpha v^2}{R} = Bqv, \text{ and therefore } v = \frac{BqR}{m_\alpha}$$

$$\therefore \mathit{Maximum\ kinetic\ energy} = E_{max} = \frac{m_\alpha}{2}\left(\frac{B^2q^2R^2}{m_\alpha^2}\right) = \frac{B^2q^2R_c^2}{8m_\alpha} \text{ --------------- (1)}$$

Here R_c is the radius of the cylinder.

Substituting the numerical values, $B = 1.2$ T, $q = 2e = 3.204 \times 10^{-19}$ C, $R_c = 0.7$ m, and $m_\alpha = 6.645 \times 10^{-27}$ kg.

$$E_{max} = 1.363 \times 10^{-12}\ \text{J} = 8.51\ \text{MeV}$$

1.3	For the equation $\frac{m_\alpha v^2}{R} = Bqv$	0.5 pts
	$E_{max} = 1.363 \times 10^{-12}\ \text{J} = 8.51\ \text{MeV}$	0.4 pts

1.4

From equation (1), it is clear that $E \propto R^2$ for particles with same charge and mass. Therefore, $\frac{E_\alpha}{E_{max}} = \left(\frac{33.26}{35.0}\right)^2$ and $E_\alpha = 7.68\ \text{MeV}$.

1.4	$E_\alpha = 7.68\ \text{MeV}$.	1.0 pts

1.5

$$\text{From } v_\alpha = \sqrt{\frac{2E_\alpha}{m_\alpha}} = \sqrt{\frac{2\times7.68\times1.602\times10^{-13}}{6.645\times10^{-27}}} = 1.92\times10^{7}\ \text{m s}^{-1} \sim 0.06c$$

1.5	For $v_\alpha = \sqrt{\frac{2E_\alpha}{m_\alpha}}$	0.5 pts
	$v_\alpha = 1.92 \times 10^7\ \text{m s}^{-1}$	1.0 pts
	$v_\alpha \sim 0.06c$ and therefore non-relativistic dynamics can be used	0.5 pts

1.6

$$E_t = E_\alpha + E_{Pb}$$

From the conservation of momentum $\vec{P}_\alpha + \vec{P}_{Pb} = 0$ and therefore, $P_\alpha^2 = P_{Pb}^2$

Using the non-relativistic equation $E = \frac{P^2}{2m}$, we have $E_{Pb} = \frac{m_\alpha}{m_{Pb}} E_\alpha$

$$\therefore E_t = \left(\frac{m_{Pb}+m_\alpha}{m_{Pb}}\right) E_\alpha = \left(\frac{210+4}{210}\right) \times 7.68\ \text{MeV} = 7.83\ \text{MeV}$$

1.6	For applying conservation of energy	0.3 pts
	For applying conservation of momentum	0.4 pts
	$E_t = 7.83\ \text{MeV}$	Q13-3

Part 2. Ionization chamber (3.5 pts)

2.1

$$Number\ of\ ion\ pairs = \frac{7.68\times10^6}{34} = 2.26 \times 10^5$$

2.1	$Number\ of\ ion\ pairs = \frac{7.68 \times 10^6}{34} = 2.26 \times 10^5$	0.5 pts

2.2

$$Sat.current = 3.7 \times 10^4 \times 2.26 \times 10^5 \times 1.602 \times 10^{-19}\ \mathrm{A} = 1.33 \times 10^{-9}\ \mathrm{A}$$

2.2	$Sat.current = 1.33 \times 10^{-9}\ \mathrm{A}$	1.5 pts

2.3

$$v_{in} = 1.33 \times 10^{-9} \times 10^6 = 1.33 \times 10^{-3}\ \mathrm{V}$$

$$v_{out} = 0.2\ \mathrm{V}$$

$$\therefore Minimum\ gain\ required = \frac{v_{out}}{v_{in}} = 150$$

2.3	$v_{in} = 1.33 \times 10^{-3}\ \mathrm{V}$	0.6 pts
	Minimum Gain = 150	1.0 pts

Q14. PHOTON INTENSITY (10 pts)

1.

At each position,

$$I(x_1) = I_0 e^{-k_A x_1}$$
$$I(x_2) = I(x_1) e^{-k_B x_2}$$
$$I_1 = I(x_2)$$
$$I(x_4) = I_1 e^{-k_B x_4}$$
$$I(x_5) = I(x_4) e^{-k_A x_5}$$
$$I_2 = I(x_5)$$

Thus,
$$I_1 = I(x_2)$$
$$= I(x_1) e^{-k_B x_2}$$
$$= I_0 e^{-k_A x_1} e^{-k_B x_2}$$
$$= I_0 \exp(-k_A x_1 - k_B x_2) \ldots\ldots\ldots\ldots\ldots(1)$$

And
$$I_2 = I(x_5)$$
$$= I(x_4) e^{-k_A x_5}$$
$$= I_1 e^{-k_B x_4} e^{-k_A x_5}$$
$$= I_1 \exp(-k_B x_4 - k_A x_5) \ldots\ldots\ldots\ldots\ldots(2)$$

From (1),

$$In \frac{I_1}{I_0} = -k_A x_1 - k_B x_2$$
$$-\frac{1}{x_1} In\left(\frac{I_1}{I_0}\right) = k_A + k_B \frac{x_2}{x_1}$$
$$k_A = -k_B \frac{x_2}{x_1} - \frac{In\left(\frac{I_1}{I_0}\right)}{x_1}$$
$$-k_A = \left[\frac{k_B x_2 + In\left(\frac{I_1}{I_0}\right)}{x_1}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Putting k_A into (2)

$$In\left(\frac{I_2}{I_1}\right) = -k_B x_4 - k_A x_5$$

$$In\left(\frac{I_2}{I_1}\right) = -k_B x_4 + [k_B x_2 + In\left(\frac{I_1}{I_0}\right)]\frac{x_5}{x_1}$$

$$In\left(\frac{I_2}{I_1}\right) - In\left(\frac{I_1}{I_0}\right)\frac{x_5}{x_1} = -k_B x_4 + \frac{k_B x_2 x_5}{x_1}$$

$$\frac{x_1 In\left(\frac{I_2}{I_1}\right) - x_5 In\left(\frac{I_1}{I_0}\right)}{x_1} = k_B[\frac{x_2 x_5 - x_1 x_4}{x_1}]$$

$$k_B = \frac{x_1 In\left(\frac{I_2}{I_1}\right) - x_5 In\left(\frac{I_1}{I_0}\right)}{x_2 x_5 - x_1 x_4}$$

Putting k_B into (3)

$$-k_A = \frac{[\frac{x_1 In\left(\frac{I_2}{I_1}\right) - x_5 In\left(\frac{I_1}{I_0}\right)}{x_2 x_5 - x_1 x_4}]x_2 + In\left(\frac{I_1}{I_0}\right)}{x_1}$$

$$k_A = \frac{x_4 In\left(\frac{I_0}{I_1}\right) - x_2 In\left(\frac{I_1}{I_2}\right)}{x_1 x_4 - x_2 x_5}$$

1. 3.5pts

$$k_A = \frac{x_4 In\left(\frac{I_0}{I_1}\right) - x_2 In\left(\frac{I_1}{I_2}\right)}{x_1 x_4 - x_2 x_5}$$

$$k_B = \frac{x_1 In\left(\frac{I_2}{I_1}\right) - x_5 In\left(\frac{I_1}{I_0}\right)}{x_2 x_5 - x_1 x_4}$$

2.

From $I_1 = I_0 e^{(-k_I w_I)}$, it is reached that $k_I = -\frac{1}{w_I}\ln\frac{I_1}{I_0}$.

Similarly, $k_{II} = -\frac{1}{w_{II}}\ln\frac{I_2}{I_1}$.

With the original forms for I_1 and I_2, it is seen that

$k_I w_I = k_A x_1 + k_B x_2$ and $k_{II} w_{II} = k_B x_4 + k_A x_5$.

Therefore, $k_I = \frac{k_A x_1 + k_B x_2}{w_I}$ and $k_{II} = \frac{k_B x_4 + k_A x_5}{w_{II}}$

or $k_I = \frac{k_A x_1 + k_B x_2}{x_1 + x_2}$ and $k_{II} = \frac{k_B x_4 + k_A x_5}{x_4 + x_5}$

2. $k_I = \frac{k_A x_1 + k_B x_2}{x_1 + x_2}$ and $k_{II} = \frac{k_B x_4 + k_A x_5}{x_4 + x_5}$ 2.0 pts

3.

Given, Taylor expansion, $f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^n(a)}{n!}(x-a)^n$

Consider the solutions,

$I_1 = I_0 e^{-(k_A x_1 + k_B x_2)}$ and $I_2 = I_1 e^{-(k_B x_4 + k_A x_5)}$.

If x_1 is about the same as x_5 and x_2 is about the same as x_4 while as I_1/I_0 and I_2/I_1 are close to 1 then the above two relations are almost identical and, therefore, cannot be solved to yield distinct values for k_A and k_B.

So, $\frac{A}{x_5 x_2 - x_2 x_5} = \frac{A}{0} = \infty$

$\ln I_1/I_0 = \ln 1 = 0$

Taylor expansion, $f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^n(a)}{n!}(x-a)^n$

In this case, two relations are used:

$$I_1 = I_0 e^{-(k_A x_1 + k_B x_2)} \qquad \text{(Eq. 1)}$$

$$\frac{I_2}{I_1} = \frac{I_1}{I_0}\frac{e^{-(k_B x_4 + k_A x_5)}}{e^{-(k_A x_1 + k_B x_2)}} \qquad \text{(Eq. 2)}$$

From Eq. 1,

$$k_A x_1 + k_B x_2 = ln\frac{I_0}{I_1} \qquad \text{(Eq. 3)}$$

From Eq. 2,

$$\frac{I_0 I_2}{I_1^2} = e^{k_A[x_1 - x_5] + k_B[x_2 - x_4]} = e^{k_A \Delta x_1 + k_B \Delta x_2}$$

$$e^{k_A \Delta x_1 + k_B \Delta x_2} = f(\Delta x_1) g(\Delta x_2)$$

$$\begin{aligned} f(\Delta x_1) &= f(0) + \Delta x_1 f'(0) \\ &= e^{k_A(0)} + k_A \Delta x_1 e^{k_A(0)} \\ &= 1 + k_A \Delta x_1 \end{aligned}$$

$$\begin{aligned} g(\Delta x_2) &= g(0) + \Delta x_2 g'(0) \\ &= e^{k_B(0)} + k_B \Delta x_2 e^{k_B(0)} \\ &= 1 + k_B \Delta x_2 \end{aligned}$$

$$e^{k_A \Delta x_1 + k_B \Delta x_2} = f(\Delta x_1) g(\Delta x_2)$$

$$= (1 + k_A \Delta x_1)(1 + k_B \Delta x_2)$$
$$= 1 + k_A k_B \Delta x_1 \Delta x_2 + k_A \Delta x_1 + k_B \Delta x_2 \approx 1 + k_A \Delta x_1 + k_B \Delta x_2$$

Where $\Delta x_1 = x_1 - x_5$ and $|\Delta x_1| \ll x_1$; and $\Delta x_2 = x_2 - x_4$ and $|\Delta x_2| \ll x_2$.

By Taylor expansion to the first order, the above equation is approximated as

$$\frac{\mathrm{I}_0 I_2}{\mathrm{I}_1^2} = [1 + k_A \Delta x_1][1 + k_B \Delta x_2] \approx 1 + \mathrm{k_A \Delta x_1} + \mathrm{k_B x_2}$$

Therefore,

$$k_A \Delta x_1 + k_B \Delta x_2 \approx \frac{I_0 I_2}{I_1^2} - 1 \qquad \text{(Eq. 4)}$$

From Eq.3, $k_A + k_B \frac{x_2}{x_1} = \frac{1}{x_1} \ln \frac{I_0}{I_1}$

From Eq. 4, $k_A + k_B \frac{\Delta x_2}{\Delta x_1} \approx \left[\frac{I_0 I_2}{I_1^2} - 1\right] \frac{1}{\Delta x_1}$

Thus,

$$k_B = \frac{\left[\frac{1}{x_1}\left(\ln \frac{I_0}{I_1}\right) - \frac{1}{\Delta x_1}\left(\frac{I_0 I_2}{I_1^2} - 1\right)\right]}{\left(\frac{x_2}{x_1} - \frac{\Delta x_2}{\Delta x_1}\right)}$$

$$k_A = \frac{1}{x_1} \ln \frac{I_0}{I_1} - k_B \frac{x_2}{x_1}$$

3.	$$k_A = \frac{1}{x_1} \ln \frac{I_0}{I_1} - k_B \frac{x_2}{x_1}$$ $$k_B = \frac{\left[\frac{1}{x_1}\left(\ln \frac{I_0}{I_1}\right) - \frac{1}{\Delta x_1}\left(\frac{I_0 I_2}{I_1^2} - 1\right)\right]}{\left(\frac{x_2}{x_1} - \frac{\Delta x_2}{\Delta x_1}\right)}$$	4.5 pts

Q15. NUCLEAR FISSION (10 pts)

1.

1.1 Velocity of fragment 1 when the time of flight is 53.928 ns

$$v_1 = \frac{\Delta x}{\Delta t_1} = \frac{0.5\ m}{53.928\times10^{-9} s} = 0.92716\ cm/ns = 9.2716 \times 10^6 ms^{-1}$$

1.1	$v_1 = 9.2716 \times 10^6 ms^{-1}$	0.5 pts

1.2 Velocity of fragment 2 when the time of flight is 34.453 ns

$$v_2 = \frac{\Delta x}{\Delta t_2} = \frac{0.5\ m}{34.453 \times 10^{-9} s} = 1.4516\ cm/ns = 1.4516 \times 10^7 ms^{-1}$$

1.2	$v_2 = 1.4516 \times 10^7 ms^{-1}$	0.5 pts

2.

Assuming the compound is static before fission, then from momentum conservation, we have;

$$m_1 v_1 - m_2 v_2 = m_{CN} v_{CN}$$

Since the compound nucleus is assumed to be static before fission, we can write the above equation in two ways. The first is that we can write

$$m_1 v_1 - m_2 v_2 = 0$$

$$m_1 v_1 = m_2 v_2$$

$$\frac{m_1}{m_2} = \frac{v_2}{v_1}$$

A different way to express these are

$$\frac{m_1 v_1 - m_2 v_2}{m_{CN}} = v_{CN}$$

$$\frac{m_1 v_1 - m_2 v_2}{m_{CN}} = 0$$

$$\frac{m_1 v_1}{m_{CN}} = \frac{m_2 v_2}{m_{CN}}$$

$$\frac{m_1 v_1}{m_{CN}} = \frac{m_2 v_2}{m_1 + m_2}$$

$$m_{CN} = \frac{m_1 v_1}{m_2 v_2}(m_1 + m_2)$$

$$m_{CN} v_2 = m_1 v_1 \left(\frac{m_1}{m_2} + 1\right)$$

Because the ratio of the fragments mass is inverse proportional to the ratio of fragments velocity,

$$m_{CN}v_2 = m_1 v_1\left(\frac{v_2}{v_1}+1\right)$$
$$m_{CN}v_2 = m_1 v_1\frac{v_2+v_1}{v_1}$$
$$m_{CN}v_2 = m_1(v_1+v_2)$$
$$m_1 = m_{CN}\frac{v_2}{v_1+v_2}$$
$$m_1 = (236.0\times 1.66\times 10^{-27}kg)\frac{1.4516\times 10^7 ms^{-1}}{0.92716\times 10^7 ms^{-1} + 1.4516\times 10^7 ms^{-1}}$$
$$m_1 = (391.76\times 10^{-27}kg)(0.6102339)$$
$$m_1 = 239.065\times 10^{-27}kg$$
$$m_1 = 239.065\times 10^{-27}\left(\frac{1}{1.66}\times 10^{27}Da\right)$$
$$m_1 = 144.015\ Da$$
$$m_1 \approx 144\ Da$$

Thus, we can also calculate,

$$m_2 = m_{CN} - m_1$$
$$m_2 = 236.0\ Da - 144.015\ Da$$
$$m_2 = 91.985\ Da$$
$$m_2 \approx 92\ Da$$

2.	$m_1 \approx 144\ Da$ $m_2 \approx 92\ Da$	3.0 pts

3.

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_1 v_2^2$$
$$K = \frac{1}{2}(144.015\times 1.66\times 10^{-27}\ kg)(9.2716\times 10^6 ms^{-1})^2 + \frac{1}{2}(91.985\times 1.66\times 10^{-27}\ kg)(1.4516\times 10^7 ms^{-1})^2$$
$$K = 1.02753\times 10^{-11}J + 1.60875\times 10^{-11}J$$
$$K = 2.63628\times 10^{-11}J$$
$$K = 2.63628\times 10^{-11}\times\frac{1\ MeV}{1.6\times 10^{-13}}$$
$$K = 164.768\ MeV$$

3.	$K = 164.768\ MeV$	1.5 pts

4.

Total excitation energy of the fragments,

$$TXE = Q - K$$
$$TXE = 190.0\ MeV - 164.768\ MeV$$
$$TXE = 25.232\ MeV$$

Neutron separation energy,

$$S_n = (939.565\ MeV/c^2 - 931.494\ MeV/c^2)c^2$$
$$S_n = 8.071\ MeV$$

Thus, the maximum possible number of neutron emission are,

$$M_n = \frac{TXE}{S_n} = \frac{25.232\ MeV}{8.071\ MeV} = 3.126\ neutrons$$

4.	$M_n = 3.126\ neutrons$	2.0 pts

5.

5.1

The charge of the first fragment,

$$Z_1 = Z_{CN}\frac{A_1}{A_{CN}}$$
$$Z_1 = 92\frac{144}{236}$$
$$Z_1 = 56.136$$
$$Z_1 \approx 56$$

The charge of the second fragment,

$$Z_2 = Z_{CN} - Z_1$$
$$Z_2 = 92 - 56$$
$$Z_2 = 36$$

5.1	$Z_1 \approx 56$ $Z_2 = 36$	1.0 pts

5.2

At full acceleration, all the energy from Coulomb repulsion will turn into kinetic energy, where

$$E_c = E_k$$
$$1.44\frac{Z_1Z_2}{R} = 164.768\ MeV$$
$$R = 1.44\frac{Z_1Z_2}{164.768}$$
$$R = 1.44\frac{56(36)}{164.768}$$
$$R = 17.620\ fm$$

5.2	$R = 17.620\ fm$	0.5 pts

5.3

Radius of fragment 1, $R_1 = 1.2\sqrt[3]{A_1} = 1.2\sqrt[3]{144} = 6.290\ fm$
Radius of fragment 2, $R_2 = 1.2\sqrt[3]{A_2} = 1.2\sqrt[3]{92} = 5.417\ fm$
Since the radius between fragments at full acceleration is 17.620 fm, the distance between the fragment surface is 5.913 fm.

5.3	The distance between the fragment surface is 5.913 fm.	1.0 pts

Q16. FISSION PRODUCT (Sm-157) (10 pts)

1.

Rate of fission = $\frac{100\ \mathrm{MW/m^3}}{3.1\times10^{-13} J} = 3.2258 \times 10^{20}$ fission s^{-1} m^{-3}

Sm-157 production = ($3.2258 \times 10^{20}\text{fission m}^{-3})(2.73 \times 10^{-5})$ atoms/fission

$$\frac{dC_0}{dt} = (8.806 \times 10^{15}) - \lambda_0 C_0$$

$$\frac{dC_0}{dt} + \lambda_0 C_0 = (8.806 \times 10^{15})$$

That has the steady state solution, $C_0 = \frac{(8.806\times10^{15})}{\lambda_0}$

The half-life of Sm-157 is 8.0 minute, thus $\lambda_0 = \frac{ln2}{480} = 0.001444\ s^{-1}$

Giving us the population of Sm-157 as $C_0 = 6.098 \times 10^{18}$

1.	Rate of fission $= 3.2258 \times 10^{20}$ fission s^{-1} m^{-3} The population of Sm-157 $= 6.098 \times 10^{18}$	2.0 pts

2.

The production rate of Eu-157,

$$\frac{dC_1}{dt} = \lambda_0 C_0 - \lambda_1 C_1$$

Where $\lambda_1$is the decay constant of Eu-157. The half-life of Eu-157 is 15.2 hours, we can express its decay constant as $\lambda_1 = \frac{ln2}{54720\ s} = 1.267 \times 10^{-5} s^{-1}$

Similarly, we can write the steady state solution of Eu-157 population as

$$C_1 = \frac{\lambda_0 C_0}{\lambda_1} = \frac{(6.098 \times 10^{18})(0.001444)}{1.267 \times 10^{-5}} = 6.950 \times 10^{20}$$

2.	Eu-157 population $= 6.950 \times 10^{20}$	1.0 pts

3.

Because Gd-157 is a neutron absorber, the rate of consumption is the reaction rate of Gd-157,

$$R = \Sigma_a \Phi$$

$$R = n\sigma_a \Phi$$

Where $\sigma_a \Phi = 240000\ (1 \times 10^{-24} cm^2) \times (7.5 \times 10^{12} n\ cm^{-2} s^{-1}) = (1.8 \times 10^{-6}) n\ s^{-1}$, thus,

$$R = n(1.8 \times 10^{-6}) s^{-1}$$

We know $n = C_2(t)$, hence,

$$\frac{dC_2}{dt} = -C_2 \sigma_a \Phi$$

Thus, we may write the full governing differential equation as,

$$\frac{dC_2}{dt} = \lambda_1 C_1 - C_2 \sigma_a \Phi$$

Giving us ,

$$C_2(t) = \frac{\lambda_1 C_1}{\sigma_a \Phi} = \frac{(1.267 \times 10^{-5})(6.950 \times 10^{20})}{(1.8 \times 10^{-6})} = 4.892 \times 10^{21}$$

3.	The steady state population of Gd-157 = 4.892×10^{21}	3.0 pts

4.

At steady state before shutdown, we can calculate that,

$$C_2(t = 0) = 4.892 \times 10^{21}.$$

After shutting down for 14 days=1,209,600 seconds, only the decay component contributes to the population of Gd-157,

$$\frac{dC_2}{dt} = -C_2 \sigma_a \Phi$$

$$C_2(t) = -C_2(0)\ e^{-\sigma_a \Phi t}$$

$$C_2(t) = -(4.892 \times 10^{21})\left[e^{-(1.8\times10^{-6})t}\right]_0^t$$

$$C_2(t = 1209600s) = -(4.892 \times 10^{21})\left[e^{-(1.8\times10^{-6})t}\right]_0^{1209600}$$

$$C_2(t = 1209600s) = (4.892 \times 10^{21})[1 - 0.11335]$$

$$C_2(t = 1209600s) = 4.337 \times 10^{21}$$

4.	The amount of Gd-157 = 4.337×10^{21}	4.0 pts

Q17. BF_3 NEUTRON DETECTION (10 pts)

1.

The energy released in a nuclear reaction can be calculated using the concept of mass-energy equivalence. In the case of the neutron capture reaction on Boron-10, using the mass-energy equivalence equation, we can convert this mass difference to energy:

$$\Delta E = \Delta m * c^2$$

By information in the table, one can calculate the released energy as flow:

$$\Delta m = (m_B + m_n) - (m_{Li} + m_\alpha)$$

$$\Delta m = (10.01294 + 1.00866) - (7.016 + 4.0026)$$

$$\Delta m = 0.003\ u$$

Now to can convert the mass in unit u to energy in unit MeV:

$$\Delta E = 0.003 \cdot 931.494 = 2.79\ MeV$$

1.	$\Delta E = 0.003 \cdot 931.494 = 2.79\ MeV$	3.0 pts

2.

Then in the case of Li ground state, the energy of Li and α will be equal to 2.79 MeV while in case of excited state which will result in gamma with energy of 0.482 MeV, Li and α will have energy equals to 2.31 MeV.

2.	2.31 MeV	2.0 pts

3.

Ground state:

From conservation of momentum law:

$$m_{Li} \cdot v_{Li} = m_\alpha \cdot v_\alpha$$
$$m_{Li}^2 \cdot v_{Li}^2 = m_\alpha^2 \cdot v_\alpha^2$$

$$\frac{1}{2} \cdot m_{Li}^2 \cdot v_{Li}^2 = \frac{1}{2} \cdot m_\alpha^2 \cdot v_\alpha^2$$

$$\frac{1}{2} \cdot m_{Li} \cdot v_{Li}^2 \cdot m_{Li} = \frac{1}{2} \cdot m_\alpha \cdot v_\alpha^2 \cdot m_\alpha$$

$$E_{Li} \cdot m_{Li} = E_\alpha \cdot m_\alpha \ \ldots\ldots. \ (1)$$

From conservation of energy law:

energy released in the reaction = fragments energies

2.79 MeV = kinetic energy of Li + kinetic energy of α

$$2.79 = E_{Li} + E_\alpha$$

$$E_{Li} = 2.79 - E_\alpha \ \ldots\ldots\ldots. \ (2)$$

By substitution of equation (2) into equation (1):

$$(2.79 - E_\alpha) \cdot m_{Li} = E_\alpha \cdot m_\alpha$$

$$2.79 \cdot m_{Li} - E_\alpha \cdot m_{Li} = E_\alpha \cdot m_\alpha$$

$$2.79 \cdot m_{Li} = E_\alpha \cdot m_\alpha + E_\alpha \cdot m_{Li}$$

$$2.79 \cdot m_{Li} = E_\alpha (m_\alpha + m_{Li})$$

$$E_\alpha = 2.79 \cdot \frac{m_{Li}}{m_\alpha + m_{Li}}$$

$$E_\alpha = 2.79 \cdot \frac{7.016}{4.0026 + 7.016}$$

$$E_\alpha = 2.79 \cdot \frac{7.016}{4.0026 + 7.016}$$

$$E_\alpha = 1.78 \ MeV$$

$$E_{Li} = 2.79 - 1.78 = 1.01 \ MeV$$

Excited state:

$$E_\alpha = (2.79 - 0.482) \cdot \frac{7.016}{4.0026 + 7.016}$$

$$E_\alpha = 1.47 \ MeV$$

$$E_{Li} = 2.31 - 1.47 = 0.84 \ MeV$$

3.	Ground state $$E_{Li} = 1.01\ MeV$$ Excited state $$E_{Li} = 0.84\ MeV$$	3.0 pts

4.

Peak (a) represents 2.31 MeV (the energy of Li and α in the excited state)

Peak (b) represents 2.79 MeV (the energy of Li and α in the ground state)

The total number of detected neutrons = 3000/(1 - 0.94) = 50000

Number of counts under peak a = 50000*0.94 = 47000

4.	47000	2.0 pts

Q18. ALARA (10 pts)

1.

The energy released in a nuclear reaction can be calculated using the concept of mass-energy equivalence. In the case of the neutron capture reaction on Boron-10, using the mass-energy equivalence equation, we can convert this mass difference to energy:

$$\Delta E = \Delta m * c^2$$

By information in the table, one can calculate the released energy as flow:

$$\Delta m = (m_B + m_n) - (m_{Li} + m_\alpha)$$

$$\Delta m = (10.01294 + 1.00866) - (7.016 + 4.0026)$$

$$\Delta m = 0.003\, u$$

Now to can convert the mass in unit u to energy in unit MeV:

$$\Delta E = 0.003 \cdot 931.494 = 2.79\, MeV$$

1.	$\Delta E = 0.003 \cdot 931.494 = 2.79\, MeV$	3.0 pts

2.

Then in the case of Li ground state, the energy of Li and α will be equal to 2.79 MeV while in case of excited state which will result in gamma with energy of 0.482 MeV, Li and α will have energy equals to 2.31 MeV.

2.	2.31 MeV	2.0 pts

3.

Ground state:

From conservation of momentum law:

$$m_{Li} \cdot v_{Li} = m_\alpha \cdot v_\alpha$$
$$m_{Li}^2 \cdot v_{Li}^2 = m_\alpha^2 \cdot v_\alpha^2$$

$$\frac{1}{2} \cdot m_{Li}^2 \cdot v_{Li}^2 = \frac{1}{2} \cdot m_\alpha^2 \cdot v_\alpha^2$$

$$\frac{1}{2} \cdot m_{Li} \cdot v_{Li}^2 \cdot m_{Li} = \frac{1}{2} \cdot m_\alpha \cdot v_\alpha^2 \cdot m_\alpha$$

$$E_{Li} \cdot m_{Li} = E_\alpha \cdot m_\alpha \ .... \ (1)$$

From conservation of energy law:

energy released in the reaction = fragments energies

2.79 MeV = kinetic energy of Li + kinetic energy of α

$$2.79 = E_{Li} + E_\alpha$$

$$E_{Li} = 2.79 - E_\alpha \ .... \ (2)$$

By substitution of equation (2) into equation (1):

$$(2.79 - E_\alpha) \cdot m_{Li} = E_\alpha \cdot m_\alpha$$

$$2.79 \cdot m_{Li} - E_\alpha \cdot m_{Li} = E_\alpha \cdot m_\alpha$$

$$2.79 \cdot m_{Li} = E_\alpha \cdot m_\alpha + E_\alpha \cdot m_{Li}$$

$$2.79 \cdot m_{Li} = E_\alpha (m_\alpha + m_{Li})$$

$$E_\alpha = 2.79 \cdot \frac{m_{Li}}{m_\alpha + m_{Li}}$$

$$E_\alpha = 2.79 \cdot \frac{7.016}{4.0026 + 7.016}$$

$$E_\alpha = 2.79 \cdot \frac{7.016}{4.0026 + 7.016}$$

$$E_\alpha = 1.78 \ MeV$$

$$E_{Li} = 2.79 - 1.78 = 1.01 \ MeV$$

Excited state:

$$E_\alpha = (2.79 - 0.482) \cdot \frac{7.016}{4.0026 + 7.016}$$

$$E_\alpha = 1.47 \ MeV$$

$$E_{Li} = 2.31 - 1.47 = 0.84 \ MeV$$

3.	Ground state $$E_{Li} = 1.01\ MeV$$ Excited state $$E_{Li} = 0.84\ MeV$$		3.0 pts

4.

Peak (a) represents 2.31 MeV (the energy of Li and α in the excited state)

Peak (b) represents 2.79 MeV (the energy of Li and α in the ground state)

The total number of detected neutrons = 3000/(1 - 0.94) = 50000

Number of counts under peak a = 50000*0.94 = 47000

4.	47000	2.0 pts

Q18. ALARA (10 pts)

1.

1.1

5mR/h / 60 min./h = 0.0833 mR/min.

0.0833 mR/min. x 10 minutes = 0.833 mR total dose.

1.1	total dose = 0.833 mR.	0.5 pts

1.2

1.0 mR / 0.0833 mR/min. = 12 minutes

*The calculated dosages would be approximations. The actual dosages may vary due to scattering and other considerations. The TLD or Film Badge should be used to determine dosage received by an individual.

1.2	12 minutes	0.5 pts

2.

2.1

$$^{131}_{53}\mathrm{I} \rightarrow \, ^{131}_{54}\mathrm{Xe} + \, ^{0}_{-1}\mathrm{e}$$

2.1	$^{131}_{53}\mathrm{I} \rightarrow \, ^{131}_{54}\mathrm{Xe} + \, ^{0}_{-1}\mathrm{e}$	1.0 pt

2.2

$$T_{1/2} = 6.01 \text{ hours}$$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{8.70} = 0.0797 \text{ days}^{-1}$$

Using $A = A_o e^{-\lambda t}$, $A = 0.05\ A_0$

$$0.05\ A_0 = A_0 e^{-(0.0797)t}$$

$$0.25 = e^{-(0.1153)t}$$

$$t = 37.6 \text{ days}$$

2.2	$t = 37.6$ days	1.0 pts

3.

$$T_{1/2} = 6.01 \text{ hours}$$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{6.01} = 0.1153$$

Using $A = A_o e^{-\lambda t}$, $A = 0.25\ A_0$

$$0.25\ A_0 = A_0 e^{-(0.1153)t}$$

$$0.25 = e^{-(0.1153)t}$$

$$t = 12.02 \text{ hours}$$

3.	$T_b = 3436.364\ s$	1.0 pts

4.

Alpha particles can be stopped by very thin shielding but have much stronger ionizing potential than beta particles, X-rays, and γ-rays. When inhaled, there is no protective skin covering the cells of the lungs, making it possible to damage the DNA in those cells and cause cancer.

4.	Alpha particles can be stopped by very thin shielding but have much stronger ionizing potential than beta particles, X-rays, and γ-rays. When inhaled, there is no protective skin covering the cells of the lungs, making it possible to damage the DNA in those cells and cause cancer.	0.5 pts

5.

5.1

The decay constant,

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{17} = 0.0408 \text{ day}^{-1}$$

The number of nuclei after 30 days,

$$N = N_o e^{-\lambda t}$$
$$N = N_o e^{-(0.0408)(30)}$$
$$N = N_o(0.294)$$

The number of nuclei decayed,

$$N_o - N = N_o - N_o(0.294)$$
$$= N_o (1 - 0.294)$$
$$= 0.706\ N_o$$

The energy released,

$$2.12 \text{ J} = (0.706\ N_o)\ (21.0 \text{ x } 10^3 \text{ eV})\left(\frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right)$$

$$N_o = 8.94 \text{ x } 10^{14} \text{ nuclei}$$

$$A = \lambda N$$
$$= (0.0408 \text{ day}^{-1})\left(\frac{1 \text{ day}}{86400 \text{ s}^{-1}}\right)(8.94 \text{ x } 10^{14} \text{ nuclei})$$
$$= 4.22 \text{ x } 10^8 \text{ Bq}$$

5.1	$A = 4.22 \text{ x } 10^8$ Bq	1.0 pts

5.2

Original mass (m) = Total mass of radioactive palladium contained

$$m = N_o \text{ x } m_{\text{one atom}}$$
$$= (8.94 \text{ x } 10^{14})(103 \text{ u})\left(\frac{1.66 \times 10^{-27} \text{ kg}}{1 \text{ u}}\right)$$
$$= 1.53 \text{ x } 10^{-10} \text{ kg}$$

5.2	Total mass = $1.53 \text{ x } 10^{-10}$ kg	1.0 pts

6.

$T_{1/2}$ = 15.0 hours
The initial activity, A_0 = 0.05 μCi

The decay constant,

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{15} = 0.0462$$

$$A = A_o e^{-\lambda t}$$
$$A = (0.05 \times 10^{-6})e^{-(0.0462)(4.5)}$$
$$= 4.06 \times 10^{-8}\text{ Ci}$$
$$= 40.6 \times 10^{-9}\text{ Ci}$$
$$= 40.6 \times \text{nCi}$$
$$= 40.6 \times 10^{3}\text{ nCi}$$

Given that 1 cm³ = of blood contains 8.00 pCi, and assume that Na-24 is uniformly dispersed throughout the blood;

$$V_{total} = \frac{40.6 \times 10^3}{8}$$

$$V_{total} = 5.08\text{ liters}$$

6.	$V_{total} = 5.08$ liters	1.5 pts

7.

The decay constant,

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{14.26} = 0.0486\text{ day}^{-1} = 5.63 \times 10^{-7}\text{ s}^{-1}$$

From the formula $A = \lambda N$,

$$N = \frac{A}{\lambda} = \frac{5.22\times10^6}{5.63\times10^{-7}} = 9.28 \times 10^{12}\text{ nuclei}$$

At t =10 days, the number of nuclei remaining;

$$N = N_o e^{-\lambda t}$$
$$= (9.28 \times 10^{12})e^{-(0.0486)(10)}$$

$= 5.71 \times 10^{12}$ nuclei

Number of nuclei decayed, $N - N_0$;

$$N - N_0 = (9.28 \times 10^{12}) - (5.71 \times 10^{12})$$
$$= 3.57 \times 10^{12}$$

Calculating the energy,

$$E = (3.57 \times 10^{12})(700 \text{ keV})(1.60 \times 10^{-16} \text{ J/kEV})$$

$$= 0.400 \text{ J}$$

7.	$E = 0.400$ J	1.0 pts

8.

Convert R/hour to mR/hour;

$$\frac{456\ R}{h} \times 1000 = 456\,000 \text{ mR/h}$$

Using the inverse square law,

$$I_1 D_1^2 = I_2 D_2^2$$

$$I_1 = 456000 \text{ mR}$$
$$D_1 = 1 \text{ m}$$
$$I_2 = 2, 5 \text{ and } 100 \text{ mR/h}$$

$$D_2 = \sqrt{\frac{(456000)(1)}{(2)}}$$
$$D_2 = 477.5 \text{ m}$$

$$D_2 = \sqrt{\frac{(456000)(1)}{(5)}}$$
$$D_2 = 301.99 \text{ m}$$

$$D_2 = \sqrt{\frac{(456000)(1)}{(100)}}$$
$$D_2 = 67.5 \text{ m}$$

8.	$D_2 = 67.5$ m	1.5 pts

Q19. POSITRON EMISSION TOMOGRAPHY (10 pts)

Part 1. Problem Subheading (7.0 pts)

1.1

For FDG:

$$\lambda = \frac{\ln(2)}{T_{1/2}}$$

$$\lambda = \frac{\ln(2)}{110\, minutes \times (60\ s/minutes)}$$

$$\lambda = 1.05 \times 10^{-4} s^{-1}$$

$$N_0 = \frac{mass}{molar\ mass} \times N_A$$

$$N_0 = \frac{10 \times 10^{-3} g}{181.15\ g/mole} \times 6.022 \times 10^{23}\ mole^{-1}$$

$$N_0 = 3.324 \times 10^{19}\ molecules$$

$$A_0 = \lambda N_0$$

$$A_0 = 1.05 \times 10^{-4}\ s^{-1} \times 3.324 \times 10^{19}\ molecules$$

$$A_0 = 3.491 \times 10^{15}\ Bq$$

1.1	$A_0 = 3.491 \times 10^{15}\ Bq$	1.0 pts

1.2

For FDG, where t = 2 hours:

$$t = 2\ hr \times (3600\ s/hr) = 7200\ s$$

$$A_t = A_0 \times e^{-\lambda t}$$

$$A_t = 3.491 \times 10^{15}\ Bq \times e^{-1.05\times10^{-4}\times7200}$$

$$A_t = 1.639\ \times 10^{15}\ Bq$$

1.2	$A_t = 1.639\ \times 10^{15}\ Bq$	1.0 pts

1.3

For FDG, where t = 1 hour:

$$N_t = N_0 \times e^{-\lambda t}$$

$$N_t = 3.324 \times 10^{19} \times e^{-1.05\times10^{-4}\times1\times3600}$$

$$N_t = 2.278 \times 10^{19}\ molecules$$

$$Rate\ of\ decay, \frac{dN}{dt} = -\lambda N$$

$$Decays\ per\ minute = \lambda N_t$$

$$Decays\ per\ minute = (1.05 \times 10^{-4}\ s^{-1} \times 60\ \ s/minute) \times 2.278 \times 10^{19}\ molecules$$

$$Decays\ per\ minute = 1.435 \times 10^{17}\ minutes^{-1}$$

1.3	$Decays\ per\ minute = 1.435 \times 10^{17}\ minutes^{-1}$	2.0 pt

1.4

For FDG, where t = 4 hours:

$$\lambda = 1.05 \times 10^{-4} s^{-1}$$

$$Total\ number\ of\ decay\ in\ 4\ hours, N_T = \int_0^4 A_0 e^{-\lambda t} dt$$

$$N_T = 3.491 \times 10^{15} \int_0^4 e^{-1.05\times10^{-4} t} dt$$

$$N_T = 3.491 \times 10^{15} \times (-\frac{e^{-1.05\times10^{-4}\times4\times3600}}{1.05 \times 10^{-4}} + \frac{e^{-1.05\times10^{-4}\times0}}{1.05 \times 10^{-4}})$$

$$N_T = 3.491 \times 10^{15} \times (-\frac{e^{-1.512} - 1}{1.05 \times 10^{-4}})$$

$$N_T = 3.491 \times 10^{15} \times 7424.109$$

$$N_T = 2.592 \times 10^{19}\ decays$$

$$Total\ energy\ deposited, E_T = N_T E_d$$

$$E_T = 2.592 \times 10^{19} \times 0.5\ \mu J$$

$$E_T = 5.184 \times 10^{12}\ J$$

1.4	$E_T = 5.184 \times 10^{12}\ J$	3.0 pts

Part 2. Problem Subheading (3.0 pts)

2.1

$$T_{eff} = 35\ minutes$$

$$\lambda_{eff} = \lambda + \lambda_b$$

$$\lambda_{eff} = \frac{\ln(2)}{T_{eff}}, \lambda = \frac{\ln(2)}{T_{1/2}}, \lambda_b = \frac{\ln(2)}{T_b}$$

$$\frac{\ln(2)}{35\ minutes \times 60\ s/minute} = \frac{\ln(2)}{90\ minutes \times 60\ s/minute} + \frac{\ln(2)}{T_b}$$

$$\frac{\ln(2)}{T_b} = 2.017 \times 10^{-4}$$

$$T_b = \frac{\ln(2)}{2.017 \times 10^{-4}}$$

$$T_b = 3436.364\ s$$

2.1	$T_b = 3436.364\ s$	2.0 pts

2.2

An advantage of using short-lived radionuclides in imaging studies is the reduction of radiation exposure to the patient. Because these radionuclides have a shorter half-life, they undergo decay relatively quickly, resulting in a shorter period of time during which the patient is exposed to radiation.

2.2	An advantage of using short-lived radionuclides in imaging studies is the reduction of radiation exposure to the patient. Because these radionuclides have a shorter half-life, they undergo decay relatively quickly, resulting in a shorter period of time during which the patient is exposed to radiation	1.0 pts

Q20. HALF-LIFE DETERMINATION (10 pts)

Part 1.

1.1

The time required for the number of radioactive nuclei to decrease to one-half the original number.

1.1	The time required for the number of radioactive nuclei to decrease to one-half the original number.	0.5 pts

1.2

Subtracting the background counts, the decay counts are

$N_1 = 372 - 5(15) = 297$ in the first 5 min interval
$N_2 = 337 - 5(15) = 262$ in the second.

1.2	$N_1 = 372 - 5(15) = 297$ $N_2 = 337 - 5(15) = 262$	in the first 5 min interval. in the second.	0.5 pts 0.5 pts

1.3

The amount of the radioactive sample at time t is $N = N_o e^{-\lambda t}$

Here we don't know N_o!!

Therefore, we will follow another approach. The number of decay counts between $(t = 0)$ and $(t = T)$ are:

$$N_1 = N_o e^{-\lambda(0)} - N_o e^{-\lambda T}$$

$$N_1 = N_o - N_o e^{-\lambda T}$$

$$N_1 = N_o\left(1 - e^{-\lambda T}\right) = 297$$

and the number of decay counts between $(t = 0)$ and $(t = 2T)$ are:

$$N_1 + N_2 = N_o e^{-\lambda(0)} - N_o e^{-\lambda(2T)}$$

$$N_1 + N_2 = N_o - N_o e^{-\lambda(2T)}$$

$$N_1 + N_2 = N_o(1 - e^{-2\lambda T}) = 297 + 262 = 559$$

Now, considering the ratio $r = \frac{N_1+N_2}{N_1}$ to eliminate (N_o) as follows:

$$\frac{N_1 + N_2}{N_1} = r = \frac{N_o(1 - N_o e^{-2\lambda T})}{N_o(1 - e^{-\lambda T})}$$

$$r = \frac{(1 - e^{-2\lambda T})}{(1 - e^{-\lambda T})} = \frac{(1 + e^{-\lambda T})(1 - e^{-\lambda T})}{(1 - e^{-\lambda T})} = (1 + e^{-\lambda T})$$

$$r - 1 = e^{-\lambda T}$$

$$(r - 1) = \left(\frac{N_1 + N_2}{N_1} - 1\right) = \left(\frac{N_2}{N_1}\right)$$

$$\leftarrow \quad \ln(r - 1) = -\lambda T = -\left(\frac{\ln 2}{T_{\frac{1}{2}}}\right) T$$

$$T_{\frac{1}{2}} = -\mathrm{T}\,\frac{\ln 2}{\ln\left(\frac{N_2}{N_1}\right)}$$

1.3	$T_{\frac{1}{2}} = -\mathrm{T}\,\frac{\ln 2}{\ln\left(\frac{N_2}{N_1}\right)}$	3.0 pt

1.4

The value of the half-life is:

$$T_{\frac{1}{2}} = -\mathrm{T}\,\frac{\ln 2}{\ln\left(\frac{N_2}{N_1}\right)} = -(5\,min) \times \frac{\ln 2}{\ln\left(\frac{262}{297}\right)} = 27.6\,min$$

1.4	$T_{\frac{1}{2}} = 27.6\,min$	0.5 pts

1.5

The smallest likely value for the half-life is then given by:

$$\ln\left(\frac{262 - 5}{297 + 5}\right) = -(5)\,\frac{\ln 2}{T_{\frac{1}{2}}} \longrightarrow$$

Gives: $\left(T_{\frac{1}{2}}\right)_{min} = 21.5\,min$

The largest value for the half-life is then given by:

$$\ln\left(\frac{262+5}{297-5}\right) = -(5)\ \frac{\ln 2}{T_{\frac{1}{2}}} \longrightarrow$$

Gives: $\left(T_{\frac{1}{2}}\right)_{max} = 38.7\ min$

Thus, the half-life is about:

$$T_{\frac{1}{2}} = \left(\frac{38.7+21.5}{2}\right) \pm \left(\frac{38.7-21.5}{2}\right)$$

$$T_{\frac{1}{2}} = 30.1 \pm 8.6$$

1.5	$T_{\frac{1}{2}} = 30.1 \pm 8.6$	2.0 pts

Q21. FUTION REACTION (10 pts)

1.

1 proton and 1 neutron.

1.	1 proton and 1 neutron.	0.5 pts

2.

- Because deuteron nucleus contains 1 proton and 1 neutron, while each Hydrogen nucleus contains only one proton.
- Because each heavy water molecule consists of two deuteron atoms which is relatively similar to to the the size of the the neutron, and hence the collisions would reduce the neutron's speed.

2.	• Because deuteron nucleus contains 1 proton and 1 neutron, while each Hydrogen nucleus contains only one proton.	0.5 pts
	• Because each heavy water molecule consists of two deuteron atoms which is relatively similar to to the the size of the the neutron, and hence the collisions would reduce the neutron's speed.	0.6 pts

3.

The deuteron and a triton are at rest before the fusion reaction, therefore, the total momentum before the reaction is zero.

Hence, the conservation of momentum requires:

$$\boldsymbol{P}_{{}_1^2H} + \boldsymbol{P}_{{}_1^3H} = \boldsymbol{P}_{{}_2^4He} + \boldsymbol{P}_{{}_0^1n}$$

$$0 = m_\alpha \mathrm{v}_\alpha + m_n \mathrm{v}_n$$

$$\frac{\mathrm{v_n}}{\mathrm{v}_\alpha} = \frac{m_\alpha}{m_n} = \frac{4.0026\ u}{1.0087\ u} = 3.968$$

3.	$T_{\frac{1}{2}} = -\mathrm{T}\,\frac{\ln 2}{\ln\left(\frac{N_2}{N_1}\right)}$	3.0 pt

4.

The deuteron and a triton are at rest before the fusion reaction, therefore, the total momentum before the reaction is zero.

Hence, the conservation of momentum requires:

$$\boldsymbol{P}_{{}_{1}^{2}H} + \boldsymbol{P}_{{}_{1}^{3}H} = \boldsymbol{P}_{{}_{2}^{4}He} + \boldsymbol{P}_{{}_{0}^{1}n}$$

$$0 = m_\alpha \mathrm{v}_\alpha + m_n \mathrm{v}_n$$

$$\frac{\mathrm{v_n}}{\mathrm{v}_\alpha} = \frac{m_\alpha}{m_n} = \frac{4.0026\,u}{1.0087\,u} = 3.968$$

4.	$\frac{\mathrm{v_n}}{\mathrm{v}_\alpha} = 3.968$	1.5 pts

5.

(a) The total energy of the neutron and the alpha particle can be calculated from the defect mass:

$$E = \Delta m \times 931.494\ MeV$$

$$E = [(2.0141 + 3.0160) - (4.0026 + 1.008)] \times 931.494\ MeV$$

$$E = 17.51\ MeV$$

$$17.51\ MeV = \frac{1}{2} m_n \mathrm{v}_n^2 + \frac{1}{2} m_\alpha \mathrm{v}_\alpha^2$$

$$E = \frac{1}{2}(1.0087\,u)\mathrm{v}_n^2 + \frac{1}{2}(4.0026\,u)\left(\frac{\mathrm{v}_n}{3.968}\right)^2$$

$$17.51\ MeV = (0.504\,u)v_n^2 + (0.127\,u)v_n^2$$

$$v_n^2 = \frac{17.51\ MeV}{0.631\ u} \times \frac{1\ u}{931.494\frac{MeV}{c^2}}$$

$$v_n = 0.173\ c\ = 5.19 \times 10^7\ m/s$$

The neutron kinetic energy is:

$$KE_n = \frac{1}{2}m_n v_n^2 = \frac{1}{2}\ (1.0087\ u\ ) \times (0.173\ c)^2\ \times \frac{1\ u}{931.494\frac{MeV}{c^2}}$$

$$KE_n = 14.1\ MeV$$

5.	$KE_n = 14.1\ MeV$	2.0 pts

6.

From the given [a] equation :

$$E_n + E_\alpha = (m_n c^2 + KE_n) + (m_\alpha c^2 + KE_\alpha)$$

We calculated $KE_n + KE_\alpha =$ $17.51\ eV\ \left(In\ part\ (d)\right).\ Then, for\ the\ moment\ we\ call$:

$$(KE_n + KE_\alpha) = k = 17.51\ MeV$$

Now we have Eq. 1 :

$$E_n + E_\alpha = m_n c^2 + m_\alpha c^2 + k \qquad [1]$$

From the conventionation of momentum: $(P_n = P_\alpha) \quad \Rightarrow \quad P_n^2\ c^2\ = P_\alpha^2\ c^2$

From the given [b] equation we c: $\quad E_n^2 - (m_n c^2)^2 = E_\alpha^2 - (m_\alpha c^2)^2$

$$E_n^2 - E_\alpha^2 = (m_n c^2)^2 - (m_\alpha c^2)^2$$

$$(E_n - E_\alpha)(E_n + E_\alpha) = (m_n c^2)^2 - (m_\alpha c^2)^2$$

Substituting Eq. [1] into the above equation:

$$(E_n - E_\alpha)(m_n c^2 + m_\alpha c^2 + k) = (m_n c^2)^2 - (m_\alpha c^2)^2$$

$$E_\alpha = E_n - \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + k)}$$

xNow substituting (E_α) from the above equation back to Eq. [1]:

$$E_n + E_n - \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + k)} = m_n c^2 + m_\alpha c^2 + k$$

$$2E_n = \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + k)} + m_n c^2 + m_\alpha c^2 + k$$

$$E_n = \frac{(m_n c^2 + m_\alpha c^2 + k)^2 + (m_n c^2)^2 - (m_\alpha c^2)^2}{2(m_n c^2 + m_\alpha c^2 + k)}$$

To find the kinetic energy of the neutron, we go back to Eq. [a]:

$$E_n = m_n c^2 + KE_n$$

Then,

$$KE_n = \frac{(m_n c^2 + m_\alpha c^2 + k)^2 + (m_n c^2)^2 - (m_\alpha c^2)^2}{2(m_n c^2 + m_\alpha c^2 + k)} - m_n c^2$$

Substituting the values of:

$$m_n c^2 = (1.0087\ u)c^2 \left(931.494 \frac{MeV}{c^2}\right) = 939.60\ MeV$$

And

$$m_\alpha c^2 = (4.002\ u)c^2 \left(931.494 \frac{MeV}{c^2}\right) = 3728.4\ MeV$$

And $k = 17.51\ eV$

$$KE_n = 14.0\ MeV$$

6.	$KE_n = 14.0\ MeV$	4.0 pts

Q22. SMOKE DETECTORS (10 pts)

Part 1. How smoke detectors work (2 pts)

1.1

Alpha emitting radioisotope is preferred than a beta or gamma emitting isotopes because of the properties of alpha particles produced.

1.1	Alpha particles are more easily absorbed by smoke particles OR Alpha particles are the most ionizing form of radiation.	0.5 pts

1.2

The production of Am-241 radioisotope from U-238 is detailed below:

1.2	$^{238}_{92}U + ^{1}_{0}n \rightarrow ^{239}_{92}U + ^{0}_{0}\gamma$ $^{239}_{92}U \rightarrow ^{239}_{93}Np + ^{0}_{-1}\beta + ^{0}_{0}\nu$ $^{239}_{93}Np \rightarrow ^{239}_{94}Pu + ^{0}_{-1}\beta + ^{0}_{0}\nu$ $^{239}_{94}Pu + ^{1}_{0}n \rightarrow ^{240}_{94}Pu + ^{0}_{0}\gamma$ $^{240}_{94}Pu + ^{1}_{0}n \rightarrow ^{241}_{94}Pu + ^{0}_{0}\gamma$ $^{241}_{94}Pu \rightarrow ^{241}_{94}Am + ^{0}_{-1}\beta + ^{0}_{0}\nu$	1.5 pts (0.25 pt each)

Part 2. Am-241 alternatives (1.5 pts)

2.1

The half-life of a radioisotope is related to its decay constant as:

$$t_{1/2} = \ln 2 / \lambda$$

Substituting the given decay constants in the equation gives:

2.1	$t_{1/2,A} = 14.9999$ days or 15 days $t_{1/2,B} = 365.0064$ days or 365 days $t_{1/2,C} = 199.9848$ days or 200 days $t_{1/2,D} = 100.0068$ days or 100 days	1.0 pt (0.25 pt each)

2.2

These radioisotopes are not suitable alternatives for Am-241 in a smoke detector because:

2.2	The **half-life of the sources will be too short** and when used it will result in **smoke detectors that cannot last for 10 years**.	0.5 pts

Part 3. Alpha particle and its interactions (2.5 pts)

3.1

The activity of the Am-241 in a smoke detector is one microcurie.

Now, $1\ microcurie = 37{,}000\ Bq$.

With 100% alpha emission, this corresponds to 37,000 alpha particles emitted per second.

3.1	37,000 alpha particles emitted per second	0.5 pts

3.2

The total energy transferred due to ionization by an alpha particle per 1 cm distance travelled can be calculated from the specific ionization and the energy required to produce one ion pair:

$$= 36\ \frac{eV}{ion\ pair} \times 34{,}000\ \frac{ion\ pair}{cm}$$

$$= 1.22 \times 10^{6}\ \frac{eV}{cm}\ or\ 1.22\ \frac{MeV}{cm}$$

The estimated range of the alpha particle in air is therefore:

$$= \frac{5.5\ MeV}{1.22\ \frac{MeV}{cm}}$$

$$= 4.5\ cm$$

3.2	$4.5\ cm$	1.0 pt

3.3

The total dose absorbed from alpha radiation in the entire shift is:

$$D = dose\ rate\ \times time$$

$$D = 300\ \frac{nGy}{h} \times 12\ h$$

$$D = 3{,}600\ nGy$$

The equivalent dose received by the worker from exposure to alpha particles can be calculated from the absorbed dose and radiation weighing factor:

$$H = D \times W$$

$$H = 3{,}600\ \times 20$$

$$H = 72{,}000\ nSv\ in\ the\ 12 - hour\ shift$$

If the exposure limit is 20 mSv per year (20 x 10^6 nSv per year), then the worker can work:

$$= \frac{20 \times 10^6\ \frac{nSv}{year}}{72{,}000\ nSv}$$

$$= 277.78\ shifts\ per\ year$$

3.3	Equivalent dose = 72,000 nSv The worker can report for 277.78 days (accept 277 OR 278 days)	1.0 pt (0.5 pt each)

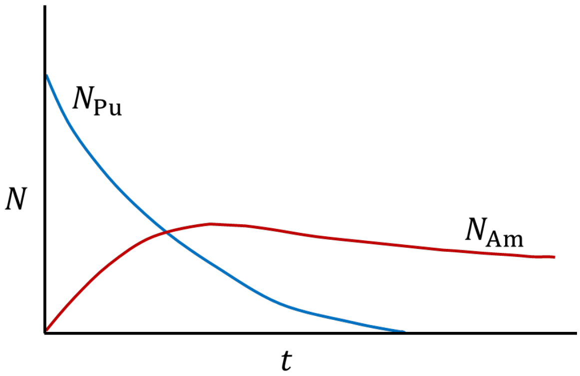
Part 4. Production of Am-241 (4 pts)

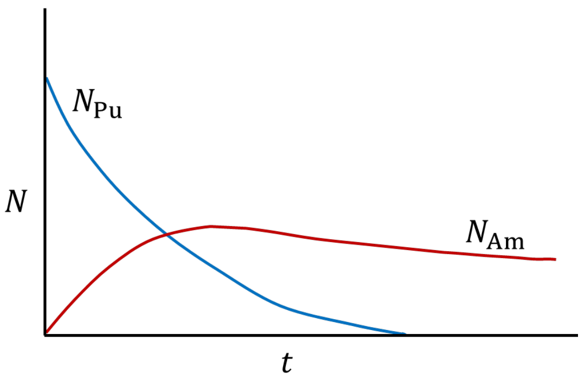
4.1

The rate equation for the buildup of Am-241 can be expressed as:

$$\frac{\mathrm{d}N_{\mathrm{Am}}}{\mathrm{d}t} = \lambda_{\mathrm{Pu}}N_{\mathrm{Pu}} - \lambda_{\mathrm{Am}}N_{\mathrm{Am}}$$

The half-life of Am-241 is 432.6 y and that of Pu-241 is 14.3 years. Based on these data and if the amount of Am-241 is zero at $t = 0$, the plot of N_{Pu} and N_{Am} as a function of time will look as follows:



4.1	Rate equation: $\frac{\mathrm{d}N_{\mathrm{Am}}}{\mathrm{d}t} = \lambda_{\mathrm{Pu}}N_{\mathrm{Pu}} - \lambda_{\mathrm{Am}}N_{\mathrm{Am}}$ Plot of amount as function of time: 	0.5 pts (0.25 pts each)

4.2

Secular equilibrium **cannot be attained**, because the daughter nuclide, Am-241 has a longer half-life (432.6 y) than its parent nuclide, Pu-241 (14.3 y). This means that Am-241 activity builds up to a maximum and then declines. Because of its shorter half-life, the parent eventually decays away and only the daughter is left.

4.2	Secular equilibrium **cannot be attained**, because the daughter nuclide, Am-241 has a longer half-life (432.6 y) than its parent nuclide, Pu-241 (14.3 y).	0.5 pts

4.3

Assuming that the decay of Am-241 is negligible, the rate equation for Am-241 can be simplified into:

$$\frac{\mathrm{d}N_{\mathrm{Am}}}{\mathrm{d}t} = \lambda_{\mathrm{Pu}} N_{\mathrm{Pu}}$$

which provides the following simplified expression for N_{Am} as a function of time:

$$N_{\mathrm{Am}} = \lambda_{\mathrm{Pu}} N_{\mathrm{Pu}} t$$

We first calculate:

$$N_{\mathrm{Pu}} = \frac{mN_A}{M} = \frac{(50\ \mathrm{g})(6.022 \times 10^{23}/\mathrm{mol})}{241.05685\ \mathrm{g/mol}} = 1.249 \times 10^{23}$$

$$\lambda_{\mathrm{Pu}} = \frac{\ln(2)}{t_{1/2}} = \frac{0.693}{14.3\ \mathrm{y}} = 0.04846\ \mathrm{y}^{-1}$$

The amount of Am-241 is then:

$$N_{\mathrm{Am}} = \lambda_{\mathrm{Pu}} N_{\mathrm{Pu}} t$$

$$N_{\mathrm{Am}} = (0.04846\ \mathrm{y}^{-1})(1.249 \times 10^{23})(0.00274\ \mathrm{y}) = 1.659 \times 10^{19}$$

4.3	$N_{\mathrm{Am}} = 1.659 \times 10^{19}$ (accept 1.658×10^{19} or 1.66×10^{19})	1.0 pt

4.4

Substituting the values provided into the following equation:

$$N_{\mathrm{Am}} = \frac{\lambda_{\mathrm{Pu}} N_{\mathrm{Pu}}}{\lambda_{\mathrm{Am}} - \lambda_{\mathrm{Pu}}} \left(\mathrm{e}^{-\lambda_{\mathrm{Pu}} t} - \mathrm{e}^{-\lambda_{\mathrm{Am}} t}\right)$$

$$N_{\mathrm{Am}} = \frac{(0.04846\ \mathrm{y}^{-1})(1.249 \times 10^{23})}{0.001602\ \ \mathrm{y}^{-1} - 0.04846\ \mathrm{y}^{-1}}\left(\mathrm{e}^{-(0.04846\ \mathrm{y}^{-1})(0.00274\mathrm{y})} - \mathrm{e}^{-(0.001602\ \mathrm{y}^{-1})(0.00274\ \mathrm{y})}\right)$$

$$N_{\mathrm{Am}} = 1.659 \times 10^{19}$$

This answer differs from the result obtained in (4.3) by 0.0069%, (or almost no difference) which means that it is reasonable to neglect the decay of Am-241 as long as the elapsed time is very short compared to its half-life.

4.4	$N_{\mathrm{Am}} = 1.659 \times 10^{19}$ (accept 1.658×10^{19} or 1.66×10^{19}) (0.5 pts) They differ by 0.0069% OR almost no difference. (0.25 pts) Yes, it is reasonable to neglect the decay of Am-241. (0.25 pts)	1.0 pt

4.5

We first obtain the decay constant of Am-241 in units of per second:

$$\lambda_{\mathrm{Am}} = 5.081 \times 10^{-11}\mathrm{s}^{-1}$$

We then calculate the activity of Am-241 as follows:

$$A_{\mathrm{Am}} = \lambda_{\mathrm{Am}} N_{\mathrm{Am}} = (5.081 \times 10^{-11}\mathrm{s}^{-1})(1.659 \times 10^{19}) = 8.43 \times 10^{8}\mathrm{Bq}$$

Each smoke detector will have $1\mu\mathrm{Ci} = 3.7 \times 10^{4}$ Bq of Am-241

We can then have $\left(\frac{8.43\times10^{8}\mathrm{Bq}}{3.7\times10^{4}\,\frac{\mathrm{Bq}}{unit}}\right)$ = 22,781 units of smoke detectors.

4.5	22,781 units of smoke detectors (accept 22,782 OR 22,780)	1.0 pt

Q23. RADIOACTIVE DATING (10 pts)

Part 1. Radioactive Dating (2 pts)

1.1

The total number of nuclei remains constant during the process of decay from parent P to daughter D:

$$P_0 = P(t_0) = P(t) + D(t)$$

The remaining number of parent nuclei after time t is given by:

$$P(t) = P_0 e^{-\lambda t}$$

Substituting the expression for P_0 gives:

$$P(t) = [P(t) + D(t)]e^{-\lambda t}$$

Solving for t,

$$t = \frac{1}{\lambda} ln\left[1 + \frac{D(t)}{P(t)}\right]$$

1.1	$t = \frac{1}{\lambda} ln\left[1 + \frac{D(t)}{P(t)}\right]$	1.0 pt

1.2

1.2	All daughter atoms are from the decay of the parent (0.5 pts) No daughter or parent atom (except for decay) is lost (0.5 pts)	1.0 pt

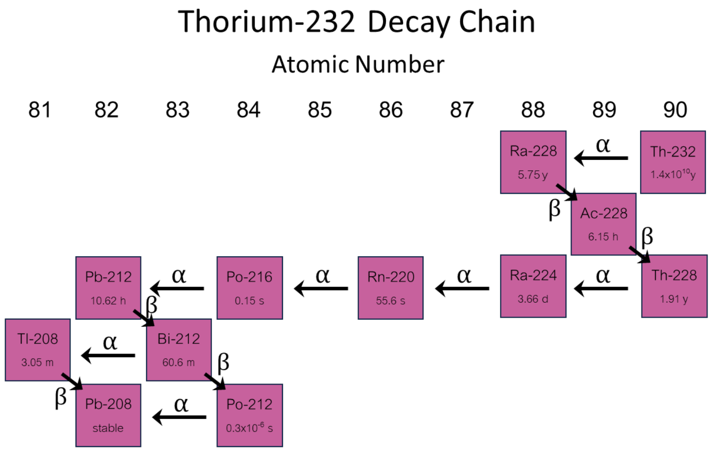
Part 2. Naturally Occurring Radioactive Materials (4.5 pts)

2.1

Decay chain for Th-232:

2.1

1.5 pts (0.375 pts each)



2.2

These radioisotopes are not suitable alternatives for Am-241 in a smoke detector because:

$$t = \frac{\boldsymbol{X}\, Pb}{U + 0.326\, Th}$$

Sum of masses: Pb206 + Pb208 + common Pb = mass of Pb

$$\left(\frac{Pb_{206}}{N_A}\right) W_{206} + \left(\frac{Pb_{208}}{N_A}\right) W_{208} + \left(\frac{p}{N_A}\right) W = Pb \qquad (1)$$

Sum number of atoms: Pb206 + Pb208 + common lead = Pb atoms

$$Pb_{206} + Pb_{208} + p = \left(\frac{Pb}{W_{Pb}}\right) N_A \qquad (2)$$

where:

Pb206 = number of Pb206 atoms
Pb208 = number of Pb208 atoms
P = number of common lead atoms (same at time = 0)
W206 = atomic weight of Pb206 (g/mole)
W208 = atomic weight of Pb208 (g/mole)
W = atomic weight of common lead (g/mole)
Pb = mass of lead (grams)
NA = Avogadro's number (atoms / mole)
WPb = atomic weight of lead (g/mole)

Multiply equation (1) with $(\frac{N_A}{W})$

$$\frac{Pb_{206} W_{206}}{W} + \frac{Pb_{208} W_{208}}{W} + p = \frac{Pb\; N_A}{W} \qquad (3)$$

Solve for p from equation (2) then substitute to equation (3)

$$\frac{Pb_{206}W_{206}}{W} + \frac{Pb_{208}W_{208}}{W} - Pb_{206} - Pb_{208} = \frac{Pb\ N_A}{W} - \frac{Pb\ N_A}{W_{Pb}}$$

Simplify

$$Pb_{206}(W - W_{206}) - Pb_{208}(W_{208} - W) = Pb\ N_A\left(\frac{W - W_{Pb}}{W_{Pb}}\right) \qquad (4)$$

Let t = age of mineral

- u_o = number of uranium atoms initially present in the mineral (t = 0)
- u = number of uranium atoms at the time of analysis
- th_o = number of thorium atoms initially present in the mineral (t = 0)
- th = number of thorium atoms at the time of analysis
- λ_u = decay constant of uranium
- λ_{th} = decay constant of thorium

Decay Formula

$$\left.\begin{aligned} u &= u_o e^{-\lambda_u t} \quad \text{which can be written as} \quad u_o = u e^{\lambda_u t} \\ th &= th_o e^{-\lambda_{th} t} \quad \text{which can be written as} \quad th_o = th\, e^{\lambda_{th} t} \end{aligned}\right\} \qquad (5)$$

Pb_{206} *and* Pb_{208} can be approximated by the decay of uranium and thorium atoms.

$$\left.\begin{aligned} Pb_{206} &= u_o - u \\ Pb_{208} &= th_o - th \end{aligned}\right\} \qquad (6)$$

Substitute (5) to (6), we get

$$\left.\begin{aligned} Pb_{206} &= u(e^{\lambda_u t} - 1) \\ Pb_{208} &= th(e^{\lambda_{th} t} - 1) \end{aligned}\right\} \qquad (7)$$

Number of atoms can also be calculated by

$$\left.\begin{aligned} u &= \frac{U}{W_u} N_A \\ th &= \frac{Th}{W_{Th}} N_A \end{aligned}\right\} \qquad (8)$$

where:

- U = mass of uranium (grams)
- Th = mass of thorium (grams)
- W_U = atomic weight of uranium isotopes (g/moles)
- W_{Th} = atomic weight of thorium isotopes (g/moles)

Substitute (8) to (7)

$$Pb_{206} = \frac{U}{W_U} N_A\left(e^{\lambda_u t} - 1\right) \qquad (9)$$

$$Pb_{208} = \frac{Th}{W_{Th}} N_A\left(e^{\lambda_{th} t} - 1\right) \qquad (10)$$

Substitute (9) & (10) to (4)

$$\frac{U}{W_U}N_A\left(e^{\lambda_u t}-1\right)(W - W_{206}) - \frac{Th}{W_{Th}}N_A\left(e^{\lambda_{th} t}-1\right)(W_{208} - W) = Pb\, N_A\left(\frac{W - W_{Pb}}{W_{Pb}}\right) \quad (11)$$

Known values are

$W_U \; = 238.029\ g/mole$
$W_{Th} = 232.0381\ g/mole$
$W = 207.2\ g/mole$
$W_{206} = 206\ g/mole$
$W_{208} = 208\ g/mole$

Substitute known values to equation (11)

$$5.041x10^{-3}\left(e^{\lambda_u t}-1\right)U - 3.448x10^{-3}\left(e^{\lambda_{th} t}-1\right)Th = Pb\left(\frac{207.2 - W_{Pb}}{W_{Pb}}\right) \quad (12)$$

For the amount of common lead in the mineral (L), from the equations (2), (9) & (10)

$$L = \frac{p}{N_A}W = \frac{\left(\frac{Pb}{W_{Pb}}\right)N_A - \frac{U}{W_U}N_A\left(e^{\lambda_u t}-1\right) - \frac{Th}{W_{Th}}N_A\left(e^{\lambda_{th} t}-1\right)}{N_A}W$$

$$L = 207.2[\frac{Pb}{W_{Pb}} - \frac{U}{238.029}\left(e^{\lambda_u t}-1\right) - \frac{Th}{232.0381}\left(e^{\lambda_{th} t}-1\right)] \quad (13)$$

Solution for equation (12) give us t, the age of mineral and (13) gives us the amount of common lead present in the mineral.

Recall Taylor Expansion

$$e^{\lambda t} - 1 = \lambda t + \frac{\lambda^2 t^2}{2} + \frac{\lambda^3 t^3}{6} + \frac{\lambda^4 t^4}{24} + \frac{\lambda^5 t^5}{120} + \cdots \ldots \ldots \ldots ..$$

Since $\lambda_u t\ or\ \lambda_{th} t < 1$ (if t < 6x10^{9}yrs.), $e^{\lambda t} - 1 \approx \lambda t_1$, then equation (12) becomes

$$5.041x10^{-3}(\lambda_u t_1)U - 3.448x10^{-3}(\lambda_{th} t_1)Th = Pb\left(\frac{207.2 - W_{Pb}}{W_{Pb}}\right)$$

Then solve for t_1

$$t_1 = \frac{Pb\left(\frac{207.2 - W_{Pb}}{W_{Pb}}\right)\frac{1}{(5.041x10^{-3})\lambda_u}}{U - \frac{3.448x10^{-3}}{5.041x10^{-3}}\left(\frac{\lambda_{th}}{\lambda_u}\right)Th} \quad (14)$$

Now let us consider equation (13) when common lead is not present

$$0 = 207.2[\frac{Pb}{W_{Pb}} - \frac{U}{238.029}\left(e^{\lambda_u t} - 1\right) - \frac{Th}{232.0381}\left(e^{\lambda_{th} t} - 1\right)]$$

Solve for t

$$t = \frac{(\frac{Pb}{W_{Pb}})(\frac{238.029}{\lambda_u})}{U + \frac{238.029}{232.0381}\left(\frac{\lambda_{th}}{\lambda_u}\right)Th}$$

Substitute all know values

W_{Pb} = 207.2 g/mole
λ_u = 1.55x$10^{-10}/yr$
λ_{th} = 4.93x$10^{-11}/yr$

Therefore

$$t = \frac{Pb\ (7{,}411{,}539{,}420)}{U + (0.326)Th}$$

$$\boldsymbol{X = 7,411,539,420}$$

2.2	$\boldsymbol{X = 7,411,539,420}$	3.0 pts

Part 3. Decay of Carbon-14 (3.5 pts)

3.1

The cosmic ray-induced reaction is

3.1	${}^{1}_{0}n + {}^{14}_{7}N \rightarrow {}^{14}_{6}C + {}^{1}_{1}p$	1.0 pt

3.2

The preserved T rex skin contains the 5 mg of C, which is equivalent to the following number of C atoms:

$$N(C\ atoms) = 5 \times 10^{-3} g \times \frac{1\ mol\ C}{12.011 g} \times \frac{6.022 \times 10^{23}\ C\ atoms}{1\ \imath}$$

$$N(C\ atoms) = 2.5069 \times 10^{20} C\ atom$$

The number of C-14 atoms can be calculated from the atomic ratio measured by AMS:

$$N(C-14\,atoms) = N(C\,atoms) \times \frac{N(C-14\,atoms)}{[N(C-12\,atoms) + N(C-14\,atoms)]}$$

Given the very low ratio of C-14 to C-12 atoms, the term $\frac{N(C-14\,atoms)}{[N(C-12\,atoms)+N(C-14\,atoms)]}$ could be approximated as $\frac{N(C-14\,atoms)}{N(C-12\,atoms)}$. The equation can therefore be simplified as:

$$N(C-14\,atoms) = N(C\,atoms) \times \frac{N(C-14\,atoms)}{N(C-12\,atoms)}$$

$$N(C-14\,atoms) = 2.5069 \times 10^{20} \times 1.7 \times 10^{-14}$$

$$N(C-14\,atoms) = 4.262 \times 10^{6} C-14\,atoms$$

The assumed C-14 activity of the sample can be calculated using $A = \lambda N$

Substituting the known values,

$$A = 1.6348 \times 10^{-6}\,Bq$$

Now, the assumed constant specific activity of C-14 in the pre-nuclear era is 227 Bq/kg.

For a 5 mg C sample, the activity would be:

$$A_0 = specific\,activity \times mass\,of\,sample$$

Substituting known values,

$$A_0 = 227\frac{Bq}{kg} \times 5mg \times \frac{1\,kg}{10^6\,mg}$$

$$A_0 = 1.135 \times 10^{-3}\,Bq$$

The age of the T rex skin could be calculated based on the radioactive decay law, and the solution is as follows:

$$t = \frac{1}{\lambda} ln\left(\frac{N_0}{N}\right)$$

$$t = \frac{1}{\lambda} ln\left(\frac{A_0}{A}\right)$$

$$t = \frac{1}{\left(\frac{\ln 2}{5{,}730\ y}\right)} ln\left(\frac{1.135 \times 10^{-3}\ Bq}{1.6348\ \times 10^{-6}\ Bq}\right)$$

$$t = 54{,}087.5\ y$$

3.2	$N(C - 14\ atoms) = 4.262 \times 10^{6}$ (1.0 pt) $t = 54{,}087.5\ y$ (1.5 pts)	2.5 pts

Q24. ELECTRON BEAM APPLICATION (10 pts)

Part 1. Inducing Chemical Reactions (5.0 pts)

1.1

EB irradiation has higher dose rates compared to gamma irradiation.

1.1	EB irradiation has **higher dose rates** thus allowing greater throughput for butanediol production.	1.0 pt

1.2

The kinetic energy of the electron can be determined from the energy of incident gamma ray: $KE_{electron} = E_{gamma} - BE$.

Substituting the known values:

$$KE_{electron} = 10 \times 10^3 eV - 530\ eV$$

$$KE_{electron} = 9{,}470\ eV$$

To calculate the total number of ethanol molecules changed by the photoelectron, we simply multiply the energy of the photoelectron with the G-value for the described process. Thus,

$$Number\ of\ ethanol\ molecules\ changed = KE_{electron} \times G - value$$

$$Number\ of\ ethanol\ molecules\ changed = 9{,}470\ eV \times \frac{4\ events}{100\ eV}$$

$$Number\ of\ ethanol\ molecules\ changed = 378.8\ OR\ 379\ ethanol\ molecules$$

1.2	378.8 OR 379 ethanol molecules	1.0 pt

1.3

To calculate the amount of power, which is equivalent to the amount of energy delivered to the ethanol per unit time during irradiation, we simply multiply the current with the energy of the accelerated electron used in the facility, assuming that all the energy is absorbed by the ethanol being irradiated. The energy absorbed per unit time in the irradiation of ethanol is:

$$Energy\ absorbed\ per\ unit\ time\ (kW) = current(mA) \times electron\ energy\ (MeV)$$

$$Energy\ absorbed\ per\ unit\ time\ (kW) = 50\ \mu A\ \times \frac{1mA}{1000\ \mu A} \times 2\ MeV$$

$$Energy\ absorbed\ per\ unit\ time\ (kW) = 0.1\ kW\ = 100\ W$$

Thus, the energy absorbed per unit time is 100 J/s.

The number of butanediol molecules in the 10 g sample can be calculated from its molar mass:

$$Number\ of\ butanediol\ molecules = \frac{mass\ butanediol}{molar\ mass\ butanediol} \times N_A$$

$$Number\ of\ butanediol\ molecules = \frac{10\ g}{90.121\ \frac{g}{mol}} \times 6.022 \times 10^{23}\ molecules/mol$$

$$Number\ of\ butanediol\ molecules = 6.682127 \times 10^{22}\ molecules$$

The energy needed to produce such number of butanediol molecules can be calculated from the G-value data:

$$Energy = number\ of\ butanediol\ molecules\ \times \left({}^{1}/_{G-value\ butanediol}\right)$$

$$Energy = 6.682127 \times 10^{22}\ molecules\ \times \frac{100\ eV}{2\ butanediol\ molecules} \times \frac{1\ J}{6.242 \times 10^{18} eV}$$

$$Energy = 535{,}255.32\ J$$

The time needed to impart this amount of energy to produce the desired amount of butanediol can be calculated from the calculated power of electrons in the EB facility:

$$Time = Energy\ \times \left({}^{1}/_{energy\ absorbed\ per\ unit\ time}\right)$$

$$Time = 535{,}255.32\ J \times \left({}^{1}\!/_{100\ \frac{J}{s}} \right)$$

$$Time = 5{,}352.5532\ s$$

$$Time = 89.2\ minutes$$

1.3	89.2 minutes	3.0 pt

Part 2. Ozone Production (5.0 pts)

2.1

We calculate first the energy delivered to air in one minute by the accelerator giving 10 W (10 J/s) of power.

$$Energy\ absorbed\ by\ air = power\ \times irradiation\ time$$

$$Energy\ absorbed\ by\ air = 10\ \frac{J}{s} \times 60\ s$$

$$Energy\ absorbed\ by\ air = 600\ J$$

From the calculated amount of energy absorbed by air, we can calculate the number of ozone molecules produced.

$$Number\ of\ ozone\ molecules\ produced$$
$$= energy\ absorbed\ by\ air \times G - value\ for\ ozone\ formation\ in\ air$$

$$Number\ of\ ozone\ molecules\ produced = 600\ J \times \frac{10\ ozone\ molecules}{100\ eV} \times \frac{6.242 \times 10^{18} eV}{1\ J}$$

$$Number\ of\ ozone\ molecules\ produced = 3.7452 \times 10^{20}\ molecules$$

And finally, the moles of ozone can be determined using Avogadro's number:

$$n_{ozone} = 6.2192 \times 10^{-4}\ mol$$

Assuming ideal gas behavior, the volume of ozone in cubic centimeters (cc) produced during the interaction of the accelerated electrons with air in the EB facility can be calculated using the ideal gas equation.

$$PV = nRT$$

$$V = \frac{nRT}{P}$$

$$V = \frac{(6.2192 \times 10^{-4}\ mol)(0.0821\ \frac{L\ atm}{mol\ K})(273.15\ K)}{1\ atm}$$

$$V = 0.013947\ L = 13.95\ cc$$

2.1	13.95 cc	3.0 pt

2.2

The total absorbed dose in the 8-hour shift is the absorbed dose per hour multiplied by the total hours of exposure:

$$absorbed\ dose\ in\ one\ shift = 45 \frac{\mu Gy}{h} \times 8h$$

$$absorbed\ dose\ in\ one\ shift = 360\ \mu Gy$$

The equivalent dose received by the worker can be calculated from the total dose absorbed by the worker and the radiation weighing factor.

$$Equivalent\ dose = absorbed\ dose \times radiation\ weighing\ factor\ for\ X - ray$$

$$Equivalent\ dose = 360\ \mu Gy \times 1$$

$$Equivalent\ dose = 360\ \mu Sv$$

If the workers are limited to 8-hour shift per day, then the number of days a worker can report to the plant can be calculated from the annual exposure limit set by the regulatory body and the equivalent dose received per day.

$$Number\ of\ days = \frac{annual\ exposure\ limit}{equivalent\ dose\ per\ day}$$

$$Number\ of\ days = \frac{20\ mSv \times \frac{1000\ \mu Sv}{1\ mSv}}{360\ \frac{\mu Sv}{day}}$$

$$Number\ of\ days = 55.6\ days$$

2.2	2.2.1. 360 μSv (1.0 pt) 2.2.2. 55.6 days (accept 55 OR 56 days) (1.0 pt)	2.0 pts

Q25. NUCLEAR SAFETY (10 pts)

1.1

The solution uses the given Wigner-Way equation below:

$$P_1(t) = 0.0622P_0[t_1^{-0.2} - (t_0 + t_1)^{-0.2}]$$

The first step is to determine the values for t_0 and t_1. The variable t_0 is the time period the reactor was operating before shutdown:

$$t_0 = 30\ days\ =\ 2{,}592{,}000\ seconds$$

The variable t_1 is the time period since the reactor has shutdown:

$$t_1 = 6\ am - 4am\ =\ 2\ hours\ =\ 7{,}200\ seconds$$

Next, the variable P_0 is found, which is the thermal power of the reactor before shutdown. Since the reactor was operating at 97% of its rated power, the rated power must be multiplied by 0.97:

$$P_0 = 0.97\ \times\ 2{,}722\ MW\text{-}thermal = 2{,}640.34\ MW\text{-}thermal$$

The above values are then substituted into the Wigner-Way equation:

$$P_1(7{,}200) = 0.0622(2{,}640.24)[7{,}200^{-0.2} - (2{,}592{,}000 + 7{,}200)^{-0.2}]$$

1.1	$P_1(7{,}200) =\ \mathbf{19.235}\ MW\text{-}thermal$	0.5 pt

1.2

Common light water reactor nuclear power plants contain three[1] primary physical barriers to the release of radioactive material to the enviro

0 Fuel cladding

1 Primary system boundary (reactor vessel and piping)

2 Containment building

[1] The fuel matrix itself it sometimes considered a fourth physical barrier.

Since the accident scenario and loss of core cooling resulting in increased fuel temperatures and fuel melting with cladding failure, only physical barriers #2 and #3 remained to prevent the release of radioactive material to the environment.

1.2	1) Primary system boundary (or reactor vessel, primary coolant system, primary system, etc.) 2) Containment (or containment building, reactor building, etc.)	0.5 pts

1.3

The following gamma ray shielding equation can be used:

$$\dot{X} = \dot{X}_0 e^{-\mu t}$$

Where,

$\dot{X}$ = Exposure rate with shield in place

$\dot{X}_0$ = Exposure rate without shield

μ = Linear attenuation coefficient

t = Thickness of shield

To reduce the exposure rate by 1,000, the following ratio is used:

$$\frac{\dot{X}}{\dot{X}_0} = \frac{1}{1000}$$

Stated another way, where μ = 29.205 cm-1

$$\frac{\dot{X}}{\dot{X}_0} = \frac{1}{1000} = e^{(-29.305t)}$$

Solving for t:

$$ln\left(\frac{1}{1000}\right) = -29.305t$$

1.3	$t = 0.236\ cm$

Q25-3

1.4

Dose can be calculated based on the source activity, the total time of exposure, and dose conversion factor, which is given as a simplified "gamma constant" in this problem. There, 1 mSv equals:

$$1\ mSv = \ gamma\ constant \times time \times activity$$

Before the mass of Xe-133 can be found, the problem must be solved for activity:

$$1\ mSv = 2.78E\text{-}5\left(\frac{mSv}{hr}\right)\left(\frac{1}{MBq}\right) \times 24hr \times activity\ (MBq)$$

$$activity = 1498.8\ MBq$$

With activity found, the specific activity value of Xe-133 can be used to convert activity to mass. However, given the units of specific activity, the activity of Xe-133 must first be converted from MBq to Ci:

$$1498.8E{+}6\ Bq \times \frac{1}{3.7E10}\left(\frac{Ci}{Bq}\right) = 0.0405\ Ci$$

Finally, the specific activity value can be used to find the amount of mass:

1.4	$\mathbf{.0405\ Ci\ /\ 1.89E5\left(\frac{Ci}{g}\right) = 2.143E\text{-}7\ g}$	0.5 pts

1.5

1.5	Evacuation can reduce (or eliminate) the amount of time that the population is exposed to radioactive material. It also increases the distance between the population and the location with radioactive material. In general, evacuation is not associated with shielding, although the distance placed between the population and the material can also be considered shielding with air/structures.	0.5 pts

Q25-4

1.6

The inverse square law can be used for this problem:

$$D_1 d_1^2 = D_2 d_2^2$$

Where,

D_1 = Dose rate at distance 1

D_2 = Dose rate at distance 2

d_1 = Distance 1

d_2 = Distance 2

The following values are given:

$$D_1 = 8.7\ mGy/hr$$

$$d_1 = 35\ cm = 0.35\ m$$

$$d_2 = 3\ m$$

Then solve for D_2

$$D_2 = \frac{D_1 d_1^2}{d_2^2}$$

$$D_2 = \frac{(8.7\ mGy/hr) \times (0.35\ m)^2}{(3\ m)^2}$$

1.6	$\boldsymbol{D_2 = 0.12\ mGy/hr}$	1.0 pts

1.7

The following gamma ray shielding equation can be used:

$$D = D_o e^{-\mu t}$$

Where,

D = Dose rate with shield in place

D_0 = Dose rate without shield

μ = Linear attenuation coefficient

t = Thickness of shield

Q25-5

The Half value thickness (HVL) is defined by:

$$HVL\ (Half - value\ thickness) = \frac{ln\,2}{\mu}$$

Therefore, μ can be found based on the following:

$$\mu = \frac{ln\,2}{HVL}$$

$$\mu = \frac{ln\,2}{12.5\ cm}$$

$$\mu = 0.056\ cm^{-1}$$

Next, solve for shielding thickness *t:*

$$D = D_o e^{-\mu t}$$

$$10\ \mu Sv/hr = 12.5e^{-0.056t}\ mSv/hr$$

$$e^{-0.056t} = \frac{10}{12.5 \times 10^3}$$

$$e^{-0.056t} = 8 \times 10^{-4}$$

$$ln\,e^{-0.056t} = ln\,8 \times 10^{-4}$$

$$-0.056t = -7.131$$

$$\boldsymbol{t = 128.485\ cm}$$

1.7	$\boldsymbol{t = 128.485\ cm}$ The reason why we need to reduce the dose rate to less than 10 μSv/h is due to the fact that 10 μSv/h is a common background dose rate caused by normal exposure to background radiation. By reducing the dose rate to below 10 μSv/h, the risk from for this particular individual will be acceptably low. This constitutes one of the best practices in radiation protection, where the time spent around a radiation source should be minimized to reduce the dose rate received.	2.0 pts

1.8

The number of atoms of Cs-137 and Cs-134 can be found using the source activity:

$$N = A/\lambda$$

Q25-6

Where,

N = Number of atoms

A = Activity

λ = Decay constant

First, convert the activity from μCi to Bq:

For Cs-137:

$$A = 156\,\mu Ci/cm^3 \times (3.7 \times 10^{10}\,Bq/Ci)$$

$$A = 5.775 \times 10^6\,Bq/cm^3$$

Next, find the decay constant λ using the half life:

$$T_{1/2} = 30.17\,years = 9.514 \times 10^8\,s$$

$$\lambda = \frac{ln\,2}{T_{1/2}}$$

$$\lambda = \frac{ln\,2}{9.514 \times 10^8\,s}$$

$$\lambda = 7.286 \times 10^{-10}\,s^{-1}$$

Lastly, solve for the number of atoms N:

$$N = A/\lambda$$

$$N = \frac{5.775 \times 10^6\,Bq/cm^3}{7.286 \times 10^{-10}\,s^{-1}}$$

$$\boldsymbol{N = 7.926 \times 10^{15}\ atoms/cm^3}$$

Repeat for Cs-134:

$$A = 26\,\mu Ci/cm^3 \times (3.7 \times 10^{10}\,Bq/Ci)$$

$$A = 9.62 \times 10^5\,Bq/cm^3$$

$$T_{1/2} = 2.06\,years = 6.496 \times 10^7\,s$$

$$\lambda = \frac{ln\,2}{T_{1/2}}$$

$$\lambda = \frac{ln\,2}{6.496 \times 10^7\,s}$$

$$\lambda = 1.067 \times 10^{-8}\,s^{-1}$$

$$N = A/\lambda$$

$$N = \frac{9.62 \times 10^5\,Bq/cm^3}{1.067 \times 10^{-8}\,s^{-1}}$$

$$\boldsymbol{N = 5.986 \times 10^{13}\ atoms/cm^3}$$

1.8	$\boldsymbol{N_{Cs137} = 7.926 \times 10^{15}\ atoms/cm^3}$	2.0 pts

$\boldsymbol{N_{Cs134} = 5.986 \times 10^{13}\ atoms/cm^3}$

1.9

The dose to each worker is the sum of the doses from individual sources:

$$D_T = \sum \dot{D}_i \times t$$

Where,

D_T = Total dose

$\dot{D}_i$ = Dose rate from source *i*

t = time

For worker A:

$$D_{137} = 10\ mSv/hr\ , D_{134} = 5\ mSv/hr$$
$$D_A = (10\, mSv/hr + 5\ mSv/hr) \times 40\ hr$$
$$\boldsymbol{D_A = 600\, mSv}$$

For worker B:

$$D_{137} = 8\ mSv/hr\ , D_{134} = 4\ mSv/hr$$
$$D_B = (8\ mSv/hr + 4\ mSv/hr) \times 40\ hr$$
$$D_B = 480\, mSv$$

For worker C:

$$D_{137} = 12\ mSv/hr\ , D_{134} = 6\ mSv/hr$$
$$D_C = (12\, mSv/hr + 6\ mSv/hr) \times 40\ hr$$
$$D_C = \boldsymbol{720\, mSv}$$

1.9	$\boldsymbol{D_A = 600\, mSv}$ $\boldsymbol{D_B = 480\, mSv}$ $\boldsymbol{D_C = 720\, mSv}$	0.5 pts

Q25-8

1.10

The activity of a source at a future time can be calculated based on the half-life:

$$A(t) = A_0 \times 0.5^{(t/T_{1/2})}$$

Where,

$\mathrm{A(t)}$ = Activity at future time t

A_0 = Current activity

t = time

$T_{\frac{1}{2}}$ = Half life

For 137-Cs:

$$A_{137}(20) = (156\,\mu Ci/cm^3) \times 0.5^{(20/30.17)}$$
$$A_{137}(20) = 98.53\ \mu Ci/cm^3$$

For 134-Cs:

$$A_{134}(20) = (26\,\mu Ci/cm^3) \times 0.5^{(20/2.06)}$$
$$A_{134}(20) = 0.031\ \mu Ci/cm^3$$

The combined activity of both isotopes after 20 years:

$$A_T(20) = A_{137}(20) + A_{134}(20)$$
$$A_T(20) = 98.53\ \mu Ci/cm^3 + 0.031\ \mu Ci/cm^3$$

1.10	$\boldsymbol{A_T(20) = 98.561\ \mu Ci/cm^3}$	1.0 pts

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See you at the

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